Exercises: Artificial Intelligence

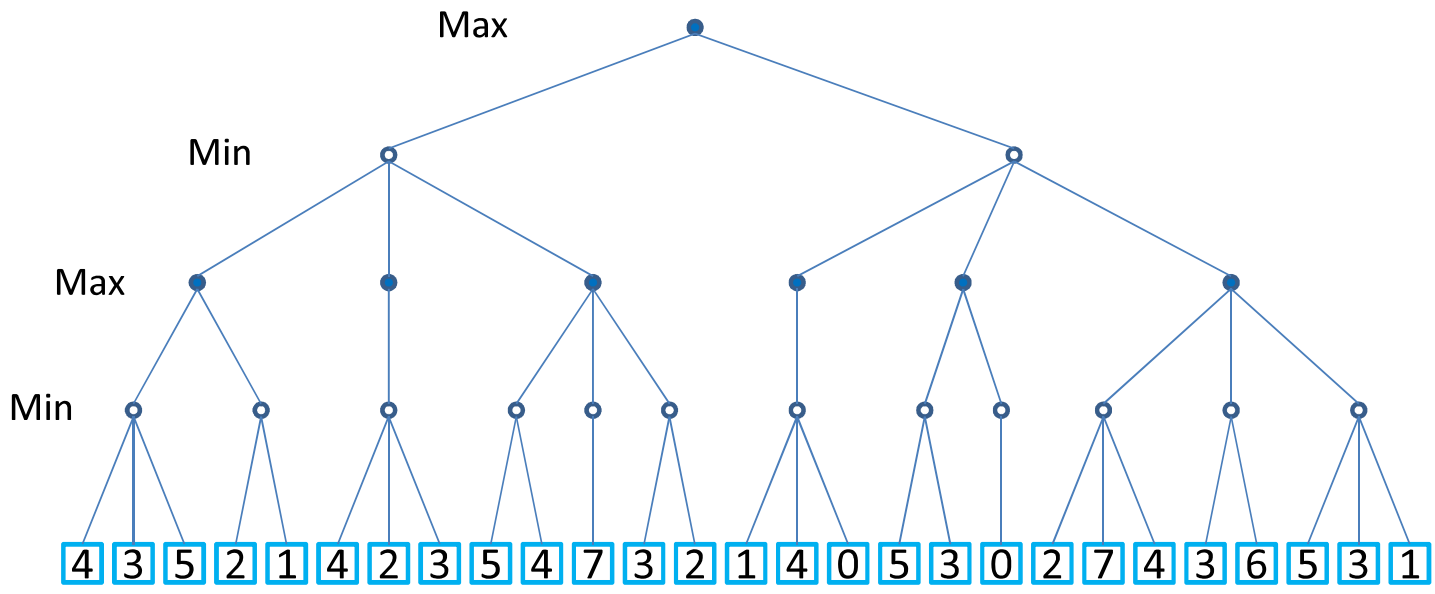
MiniMax & Constraint Processing: MiniMax Algorithm

MiniMax & Constraint Processing: MiniMax Algorithm

PROBLEM 1

Problem 1

- Perform the minimax algorithm on the figure below. First without, later with $\alpha\beta$-pruning.

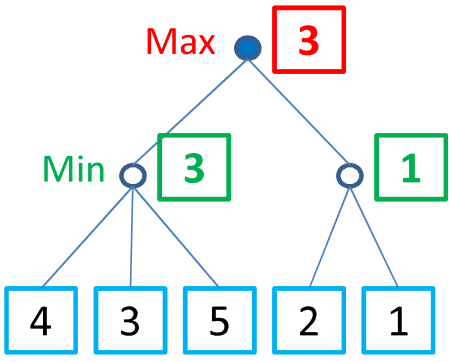


MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX ALGORITHM

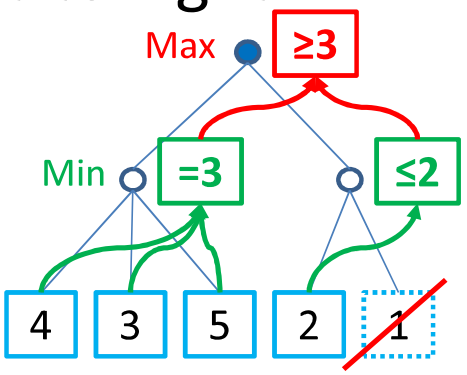
MiniMax Algorithm

- Restrictions
  - 2 players: Max = Computer & Min = Opponent
  - Deterministic, perfect information
- Depth-bound & Evaluation function
  - Construct tree (depth-bound)
  - Compute evaluation leaves
  - Propagate upwards (min/max)



$\alpha\beta$-Pruning

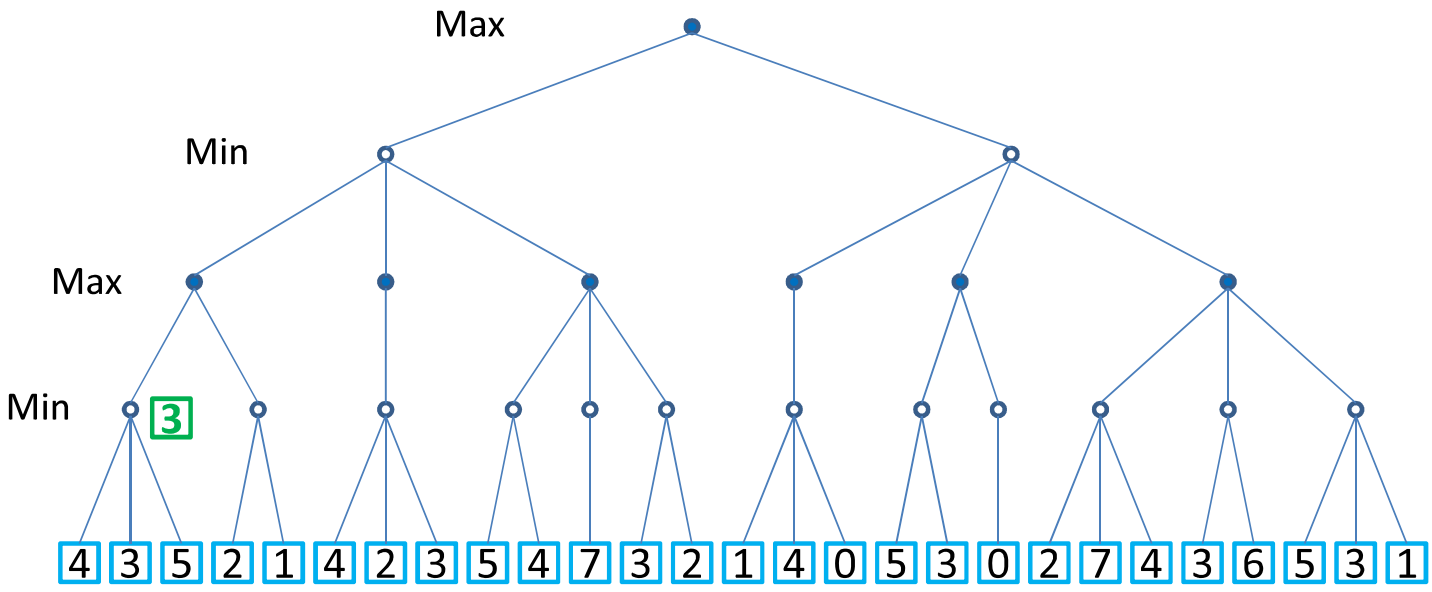
- Generally applied optimization
  - Instead of generating, then propagating
  - Interleave generation and propagation
    - Obtain information on redundant parts
- Generate tree: depth-first & Left-to-right
  - Propagate values of nodes
  - Estimates for parent nodes



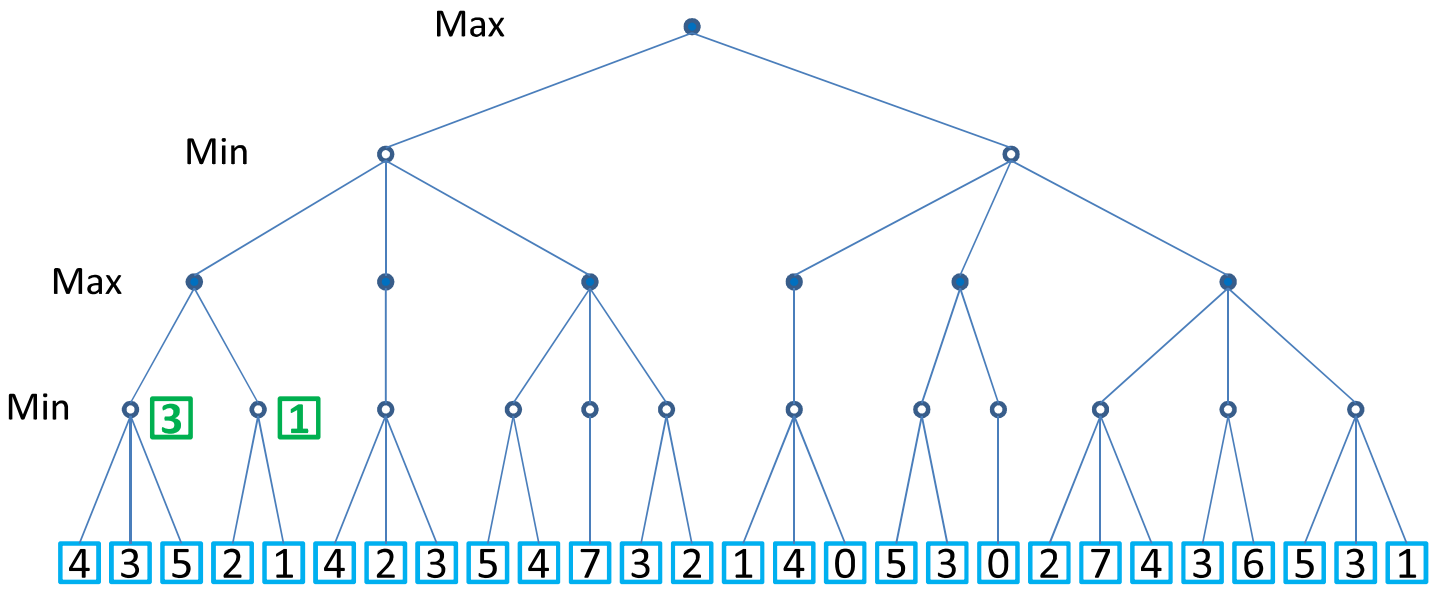
MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX WITHOUT $\alpha\beta$-PRUNING

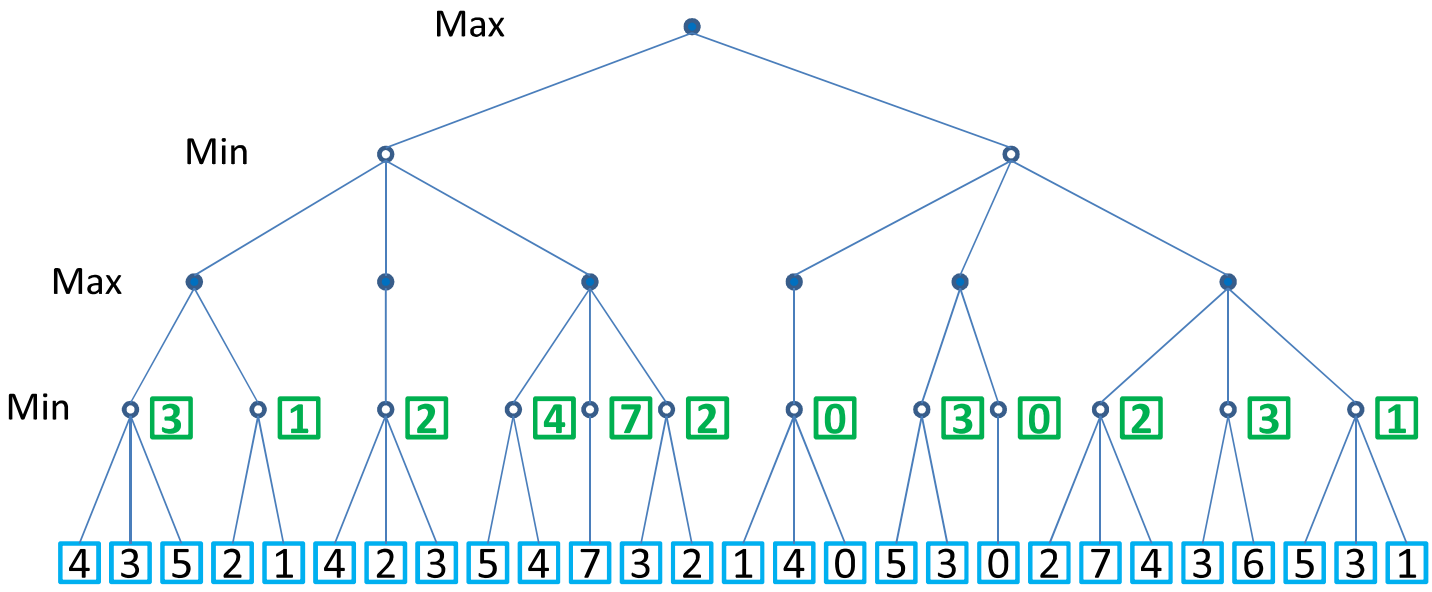
MiniMax without $\alpha\beta$-pruning



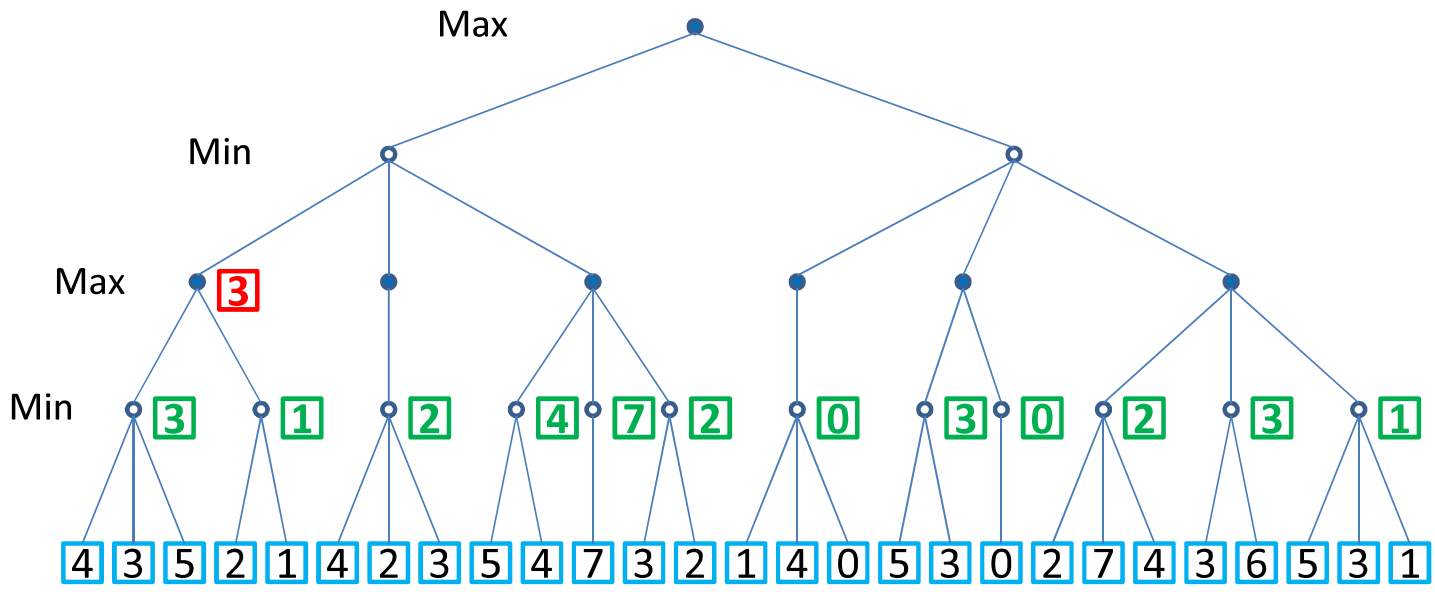
MiniMax without $\alpha\beta$-pruning



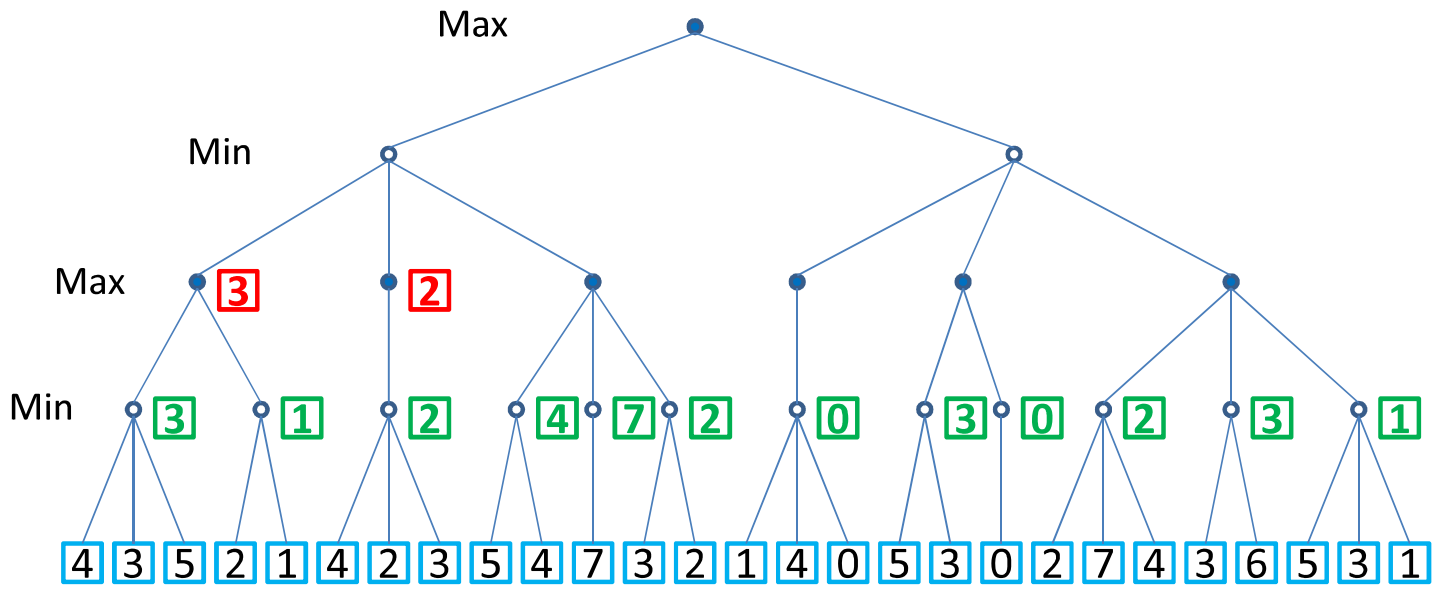
MiniMax without $\alpha\beta$-pruning



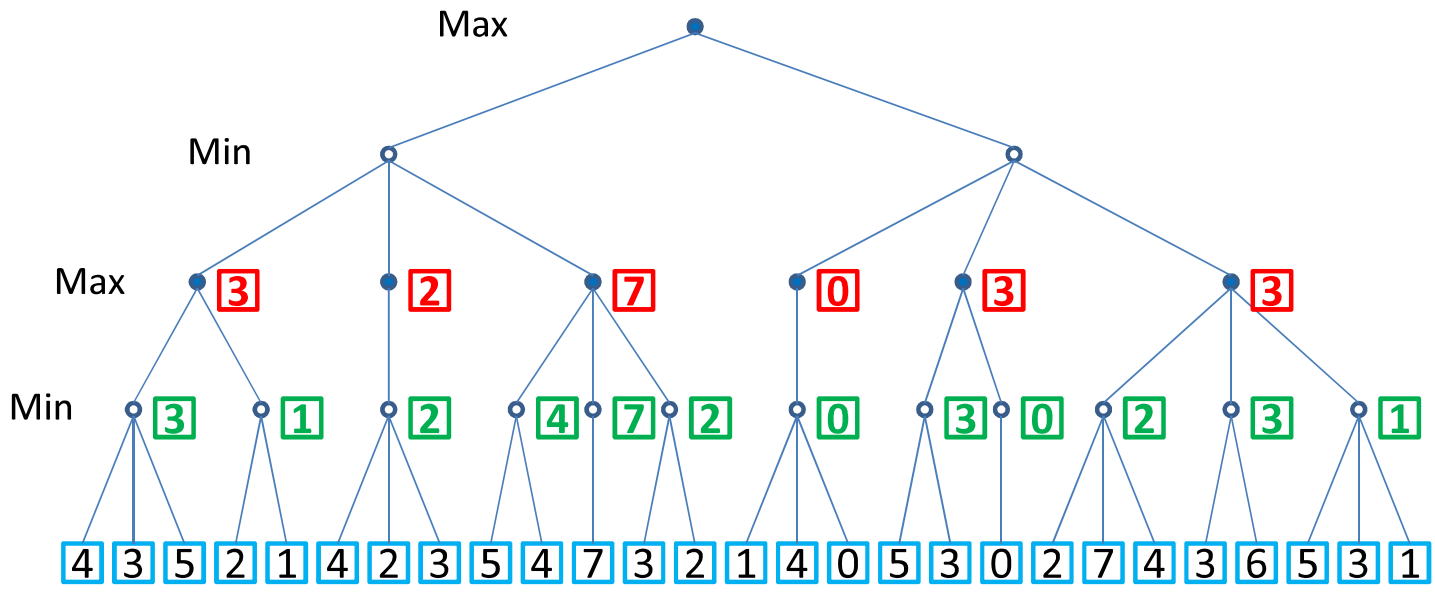
MiniMax without $\alpha\beta$-pruning



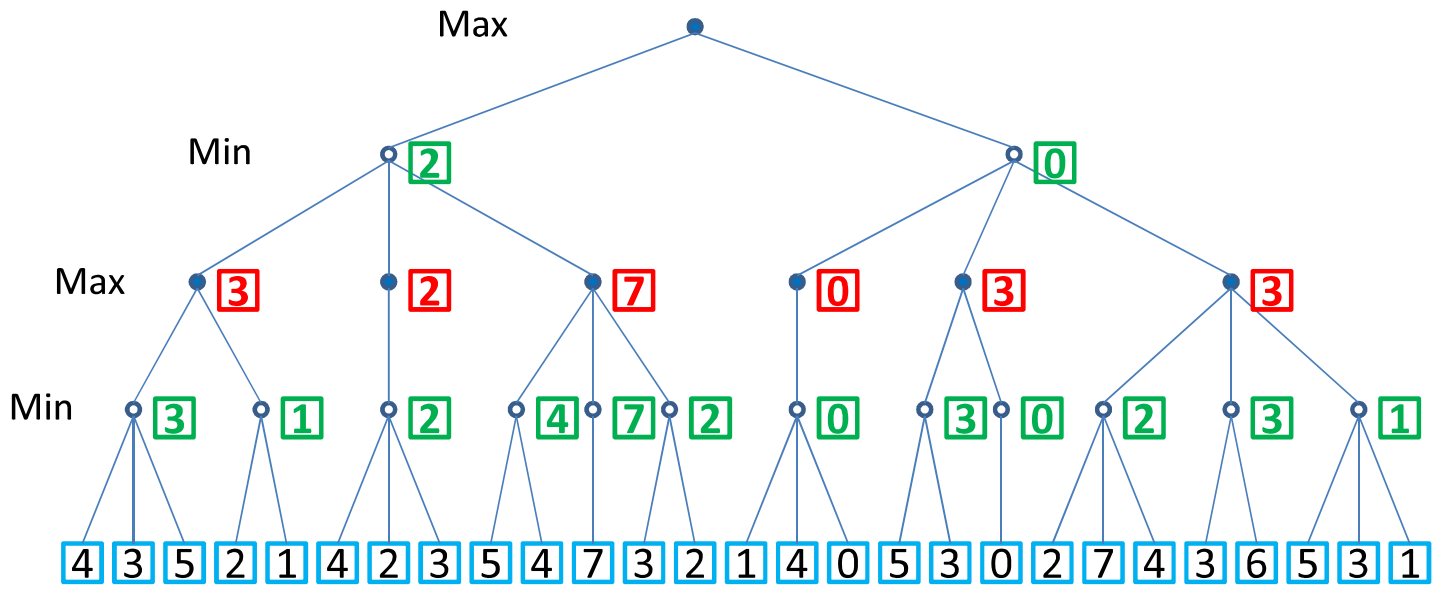
MiniMax without $\alpha\beta$-pruning



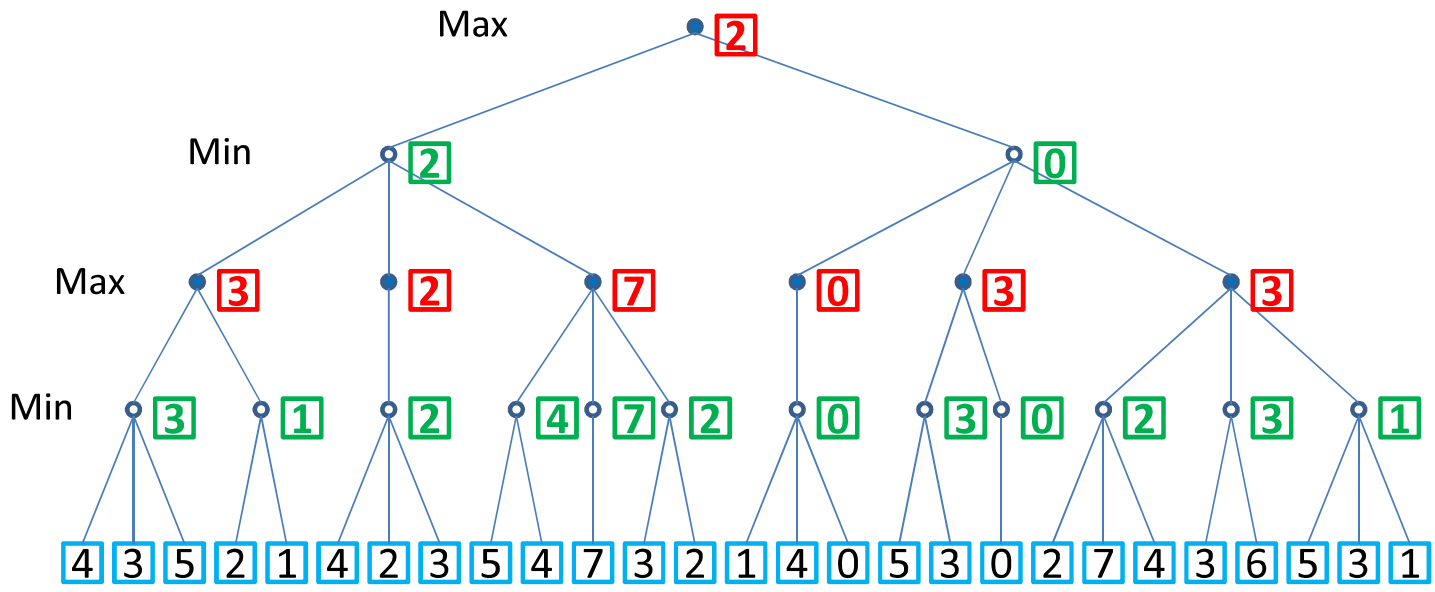
MiniMax without $\alpha\beta$-pruning



MiniMax without $\alpha\beta$-pruning



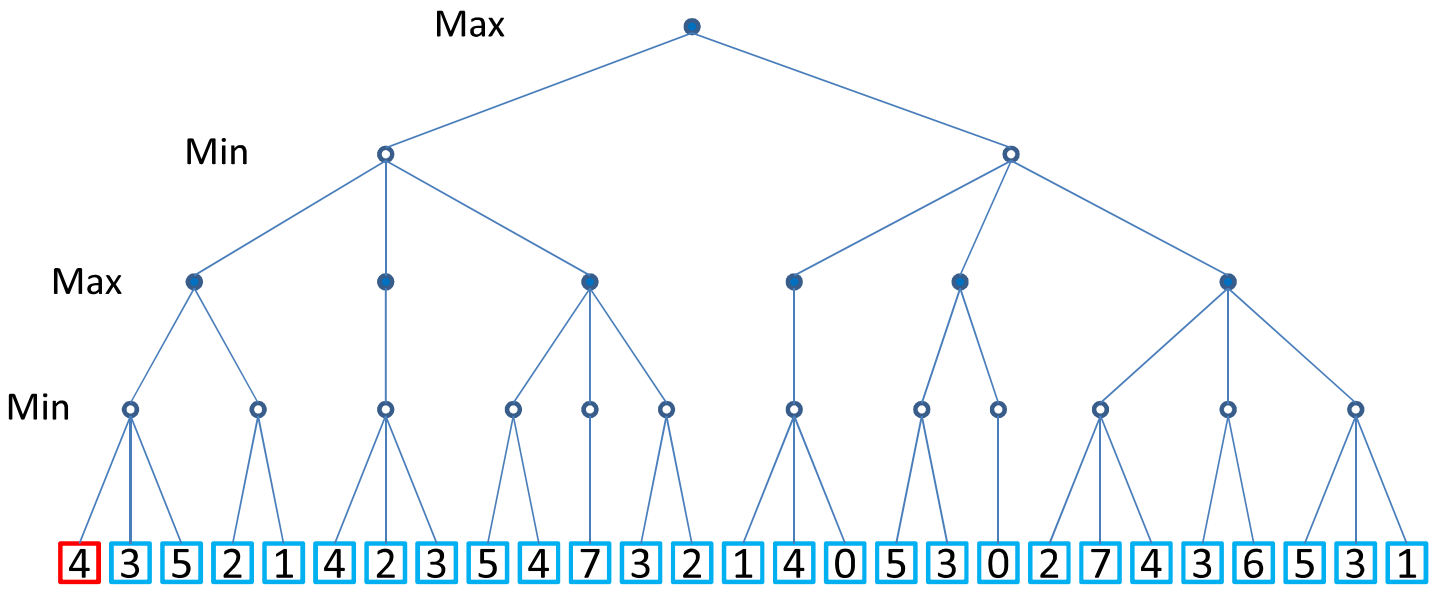
MiniMax without $\alpha\beta$-pruning



MiniMax & Constraint Processing: MiniMax Algorithm

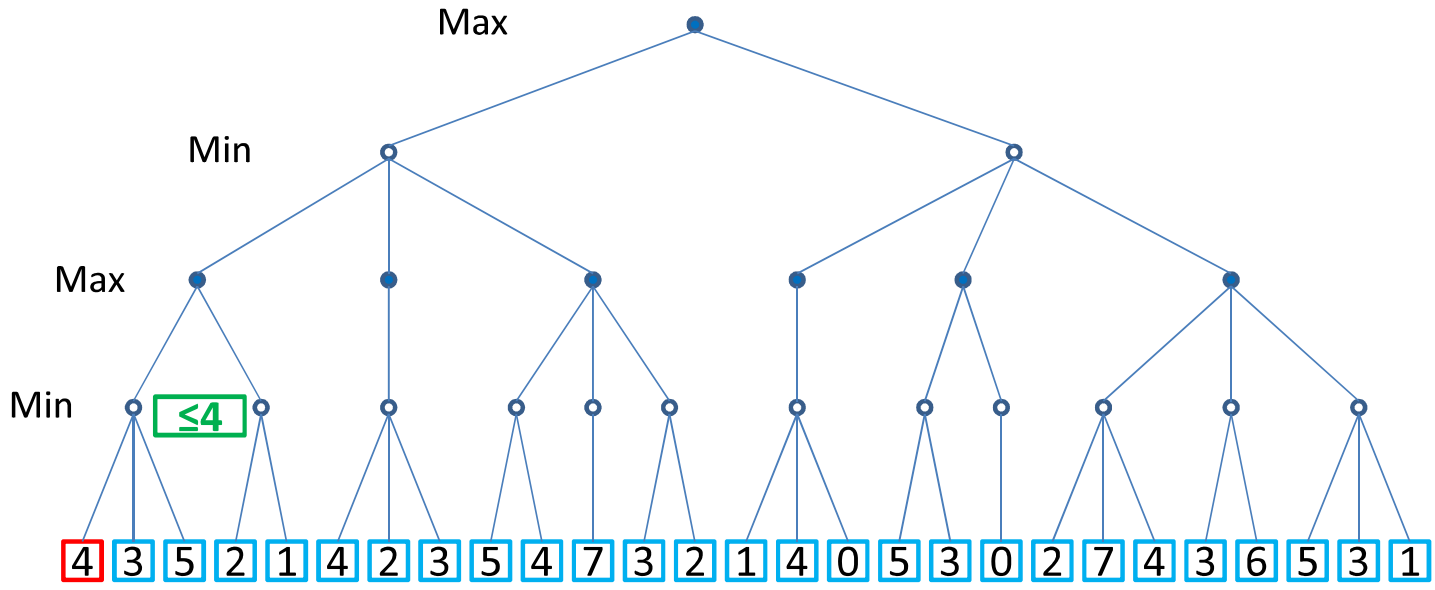
MINIMAX WITH $\alpha\beta$-PRUNING

MiniMax with $\alpha\beta$-pruning

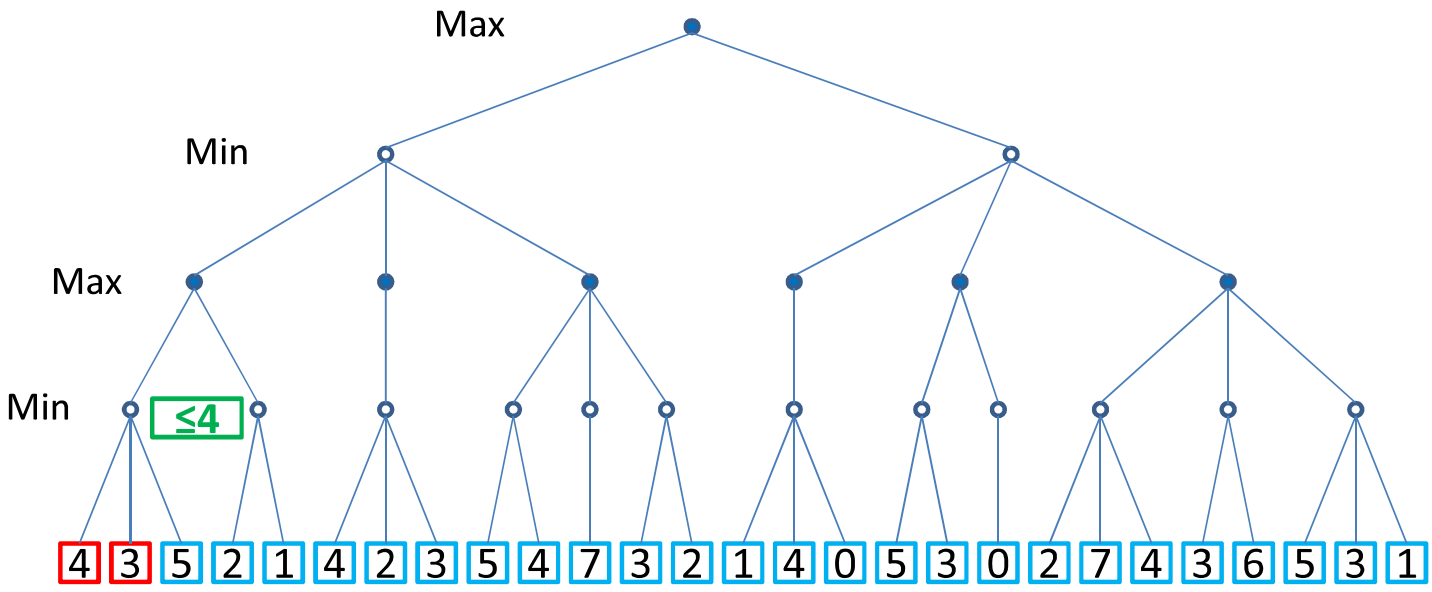


MiniMax with $\alpha\beta$-pruning

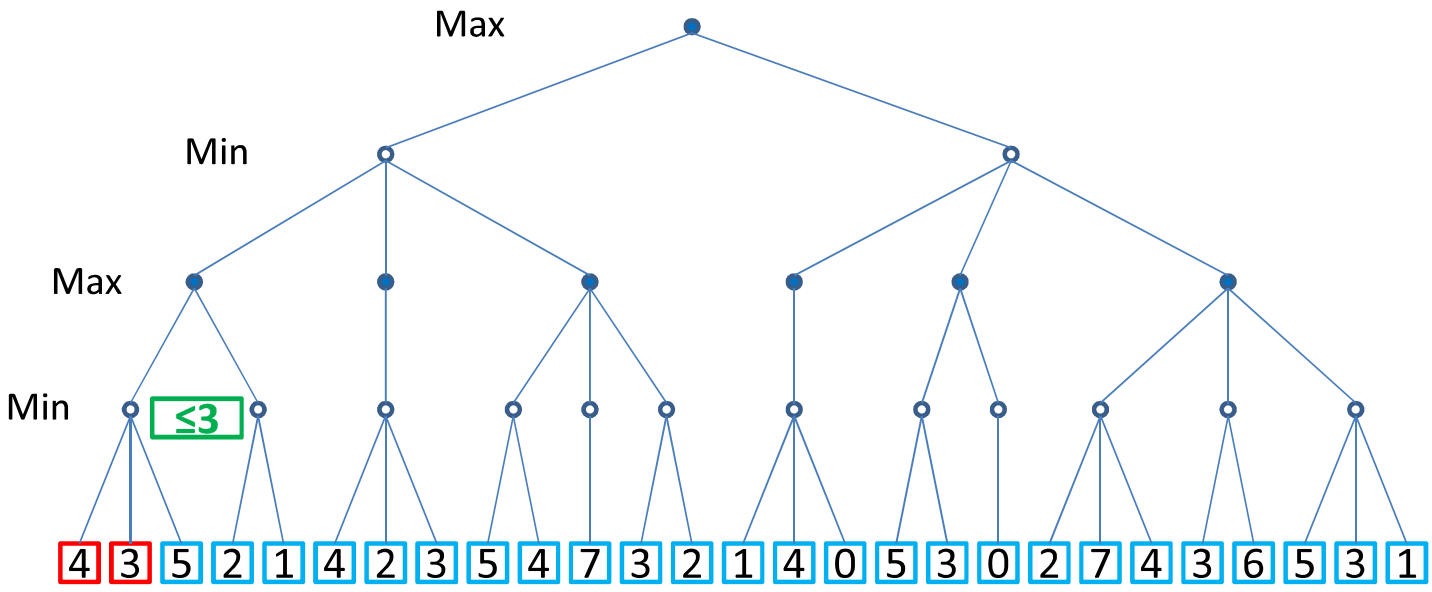
- **α-nodes**: Temporary values at MIN-nodes



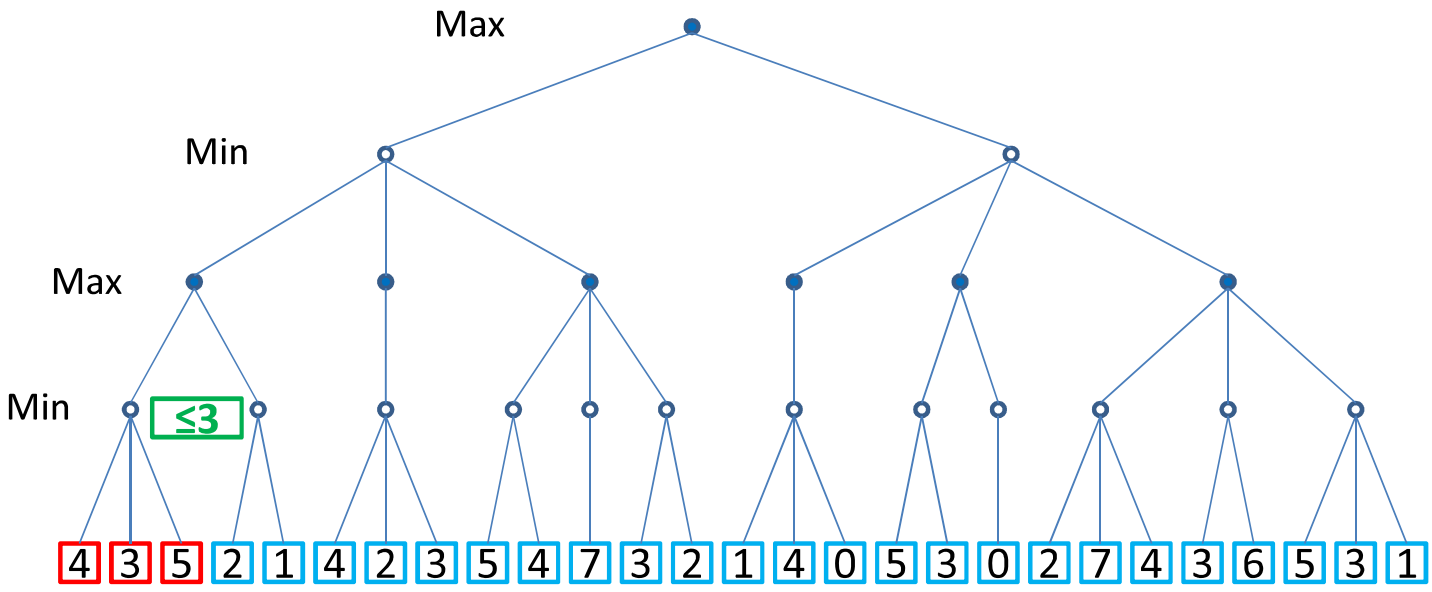
MiniMax with $\alpha\beta$-pruning



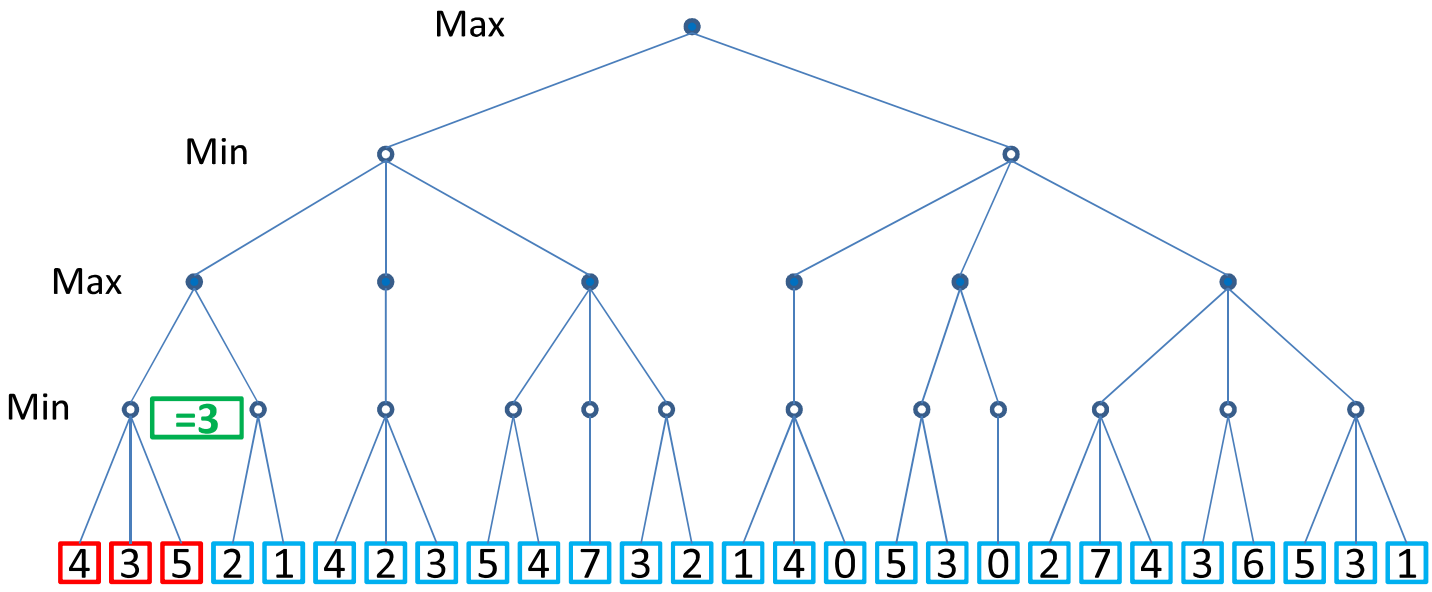
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning

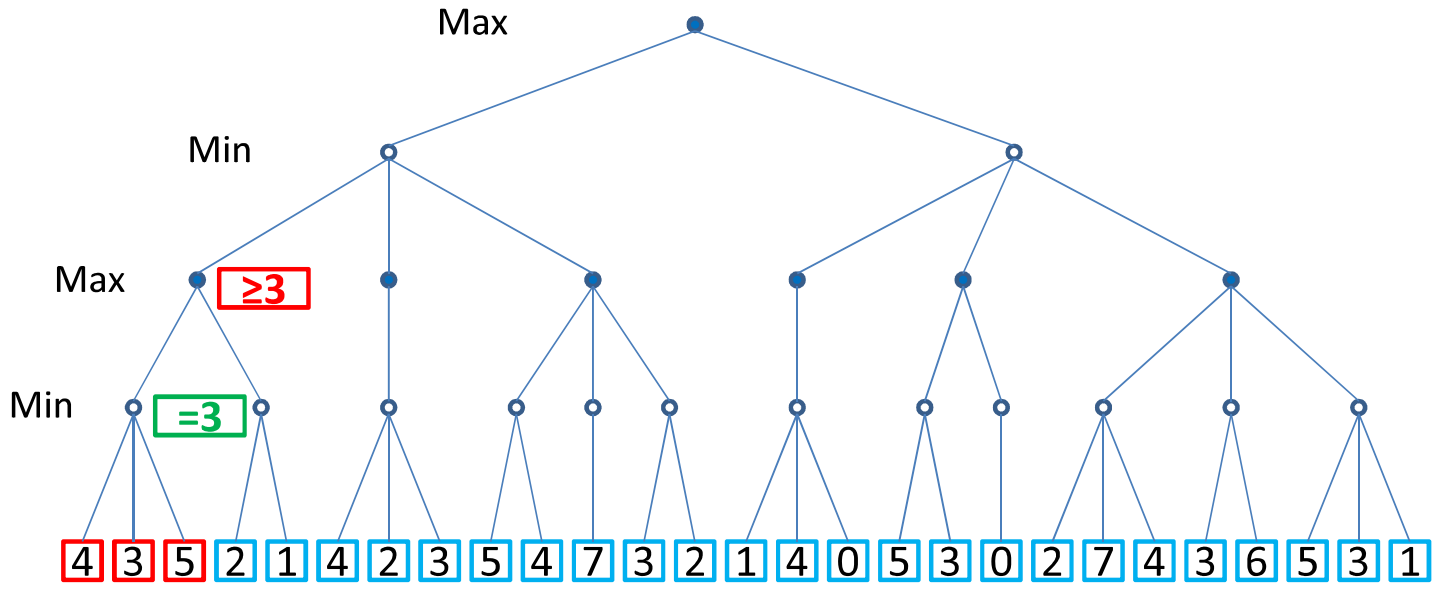


MiniMax with $\alpha\beta$-pruning

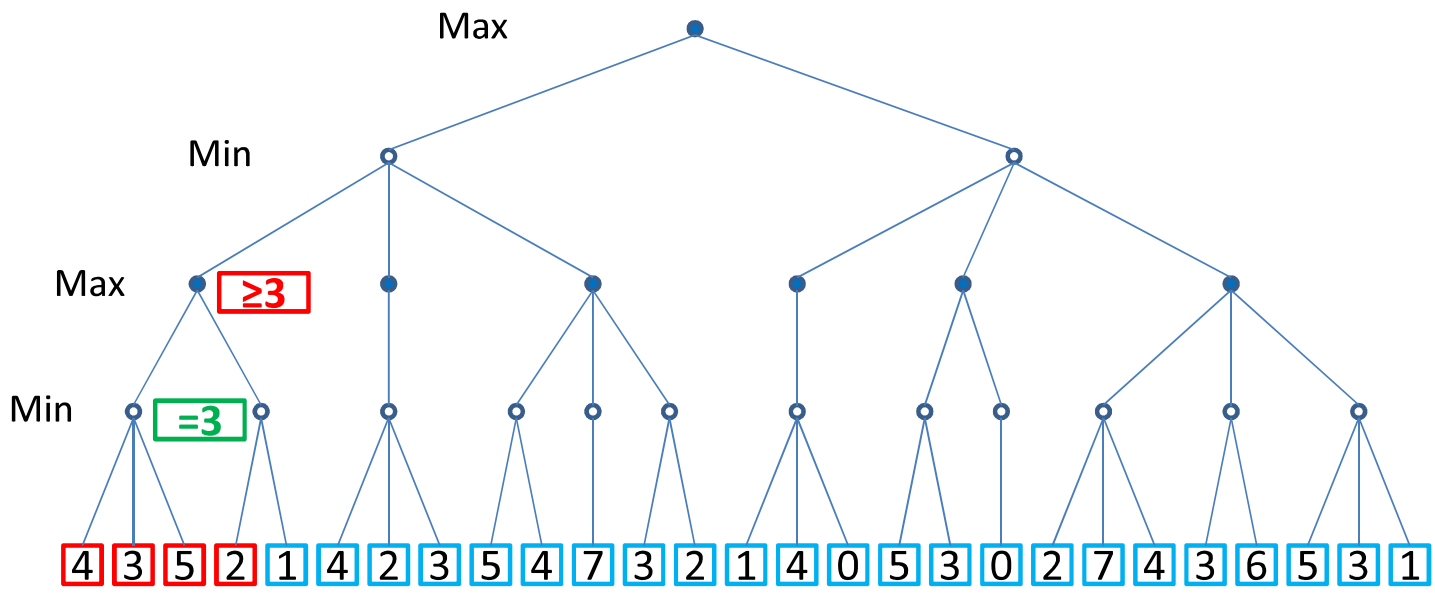


MiniMax with $\alpha\beta$-pruning

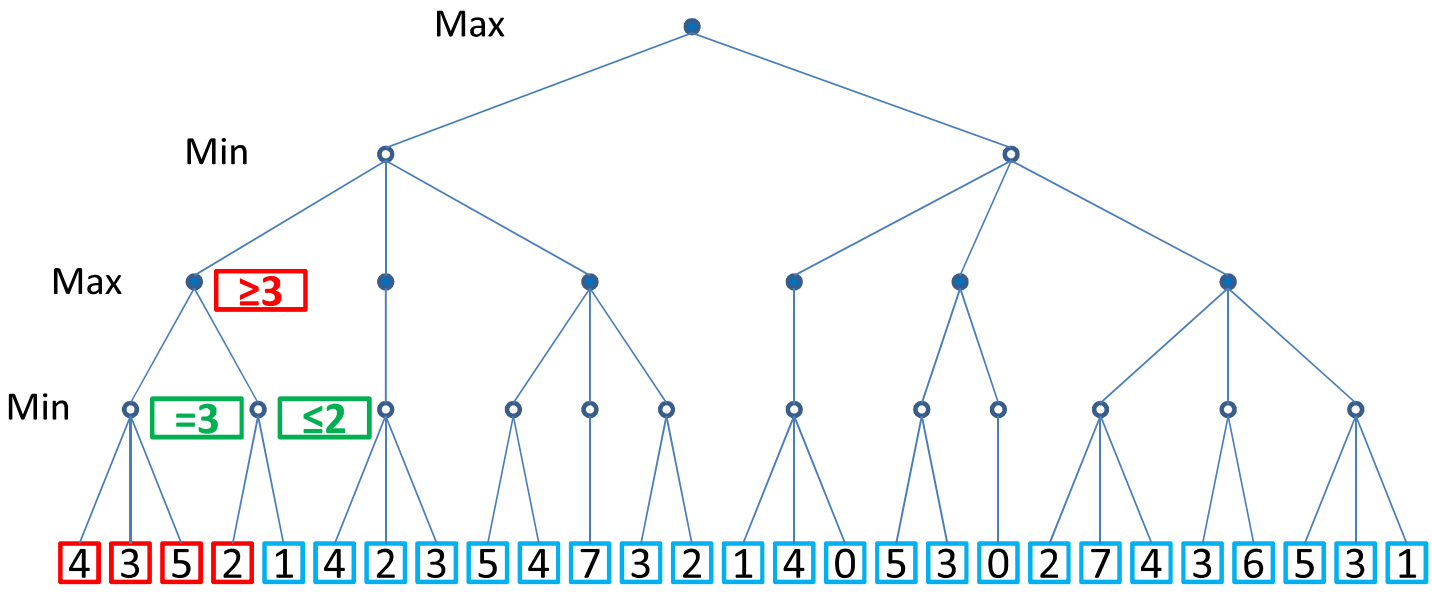
- **β-nodes**: Temporary values at MAX-nodes



MiniMax with $\alpha\beta$-pruning

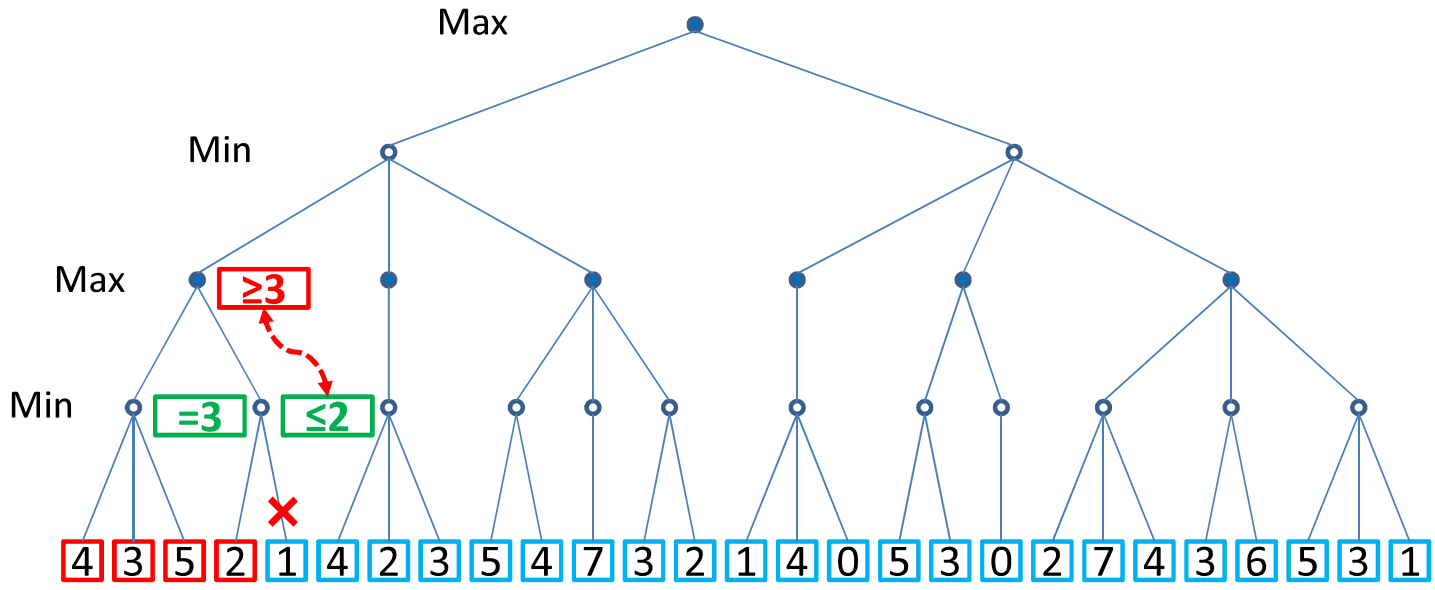


MiniMax with $\alpha\beta$-pruning

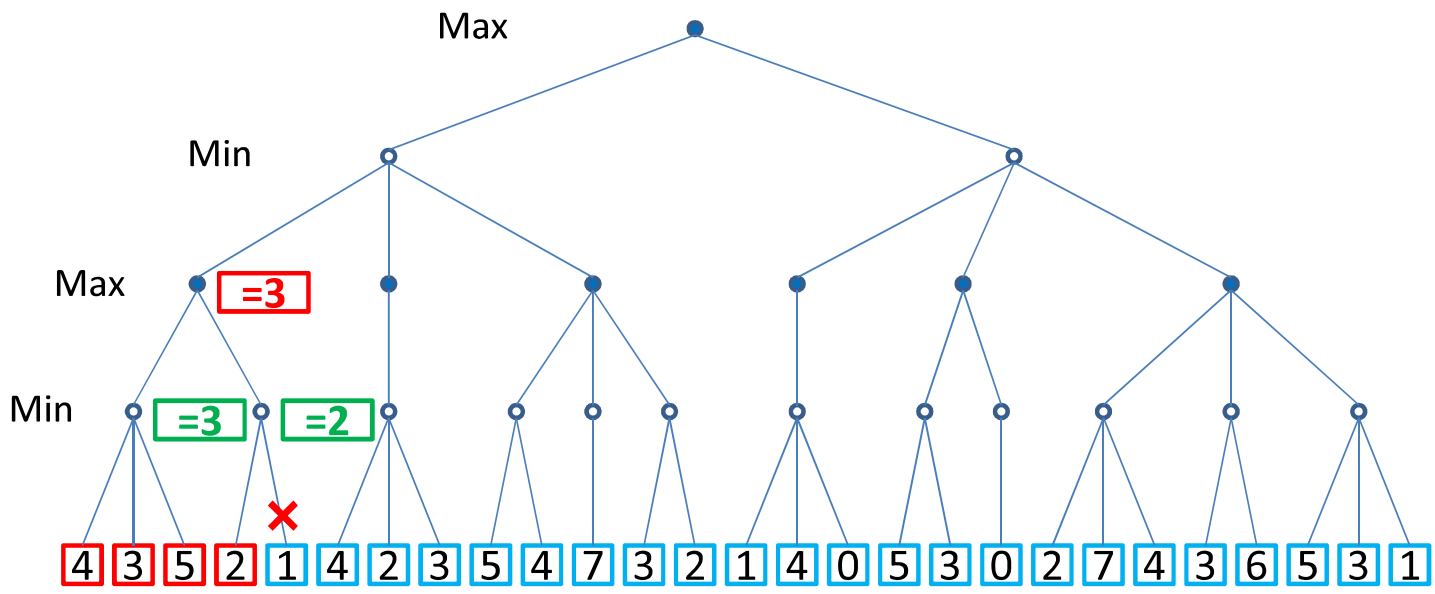


MiniMax with $\alpha\beta$-pruning

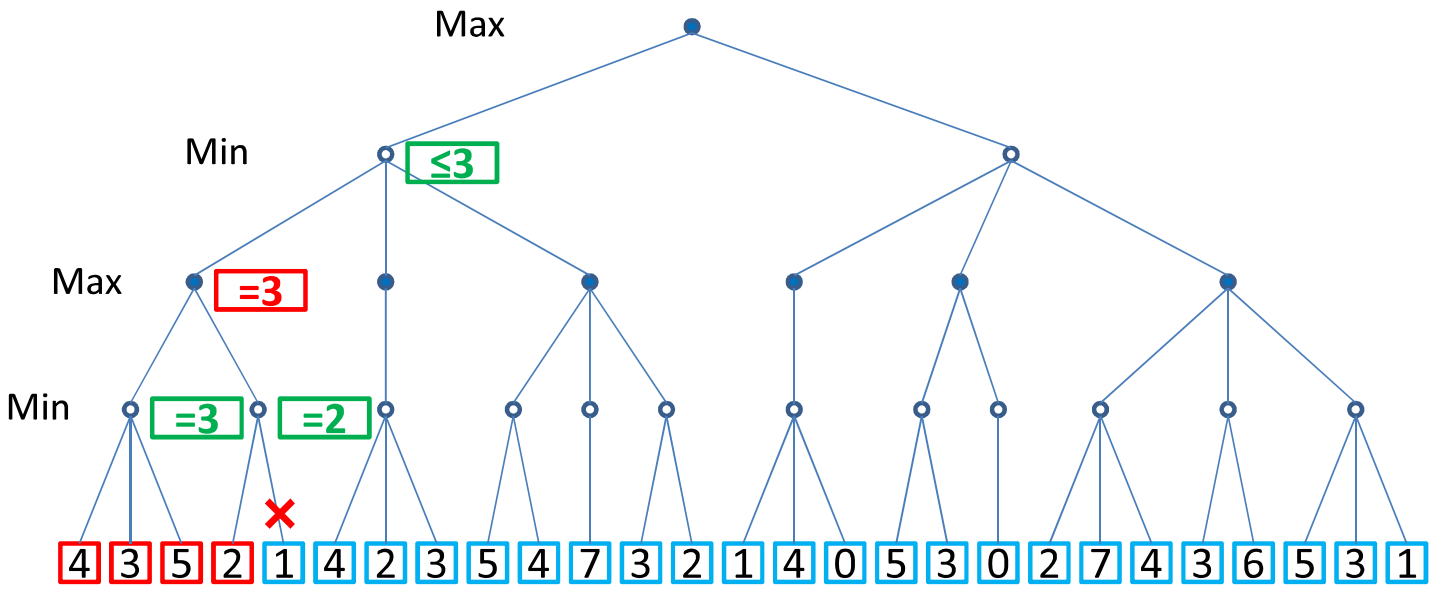
- **Prune: Parent β-node ≥ Child α-node**



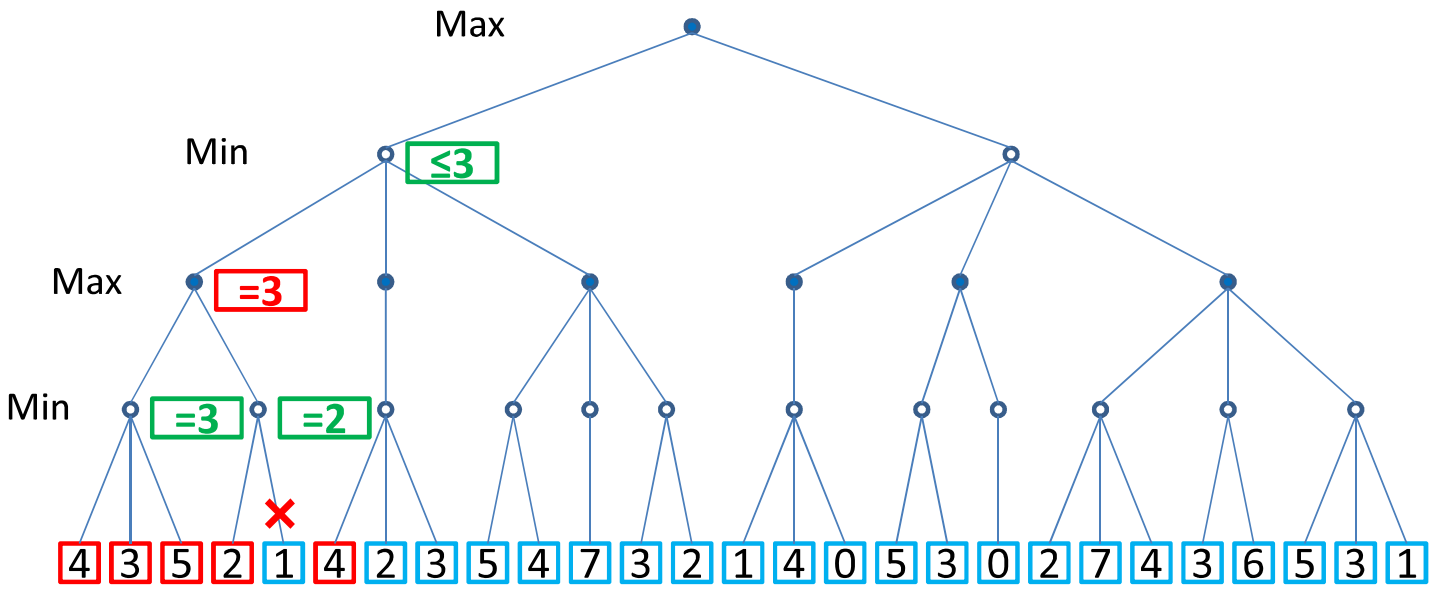
MiniMax with $\alpha\beta$-pruning



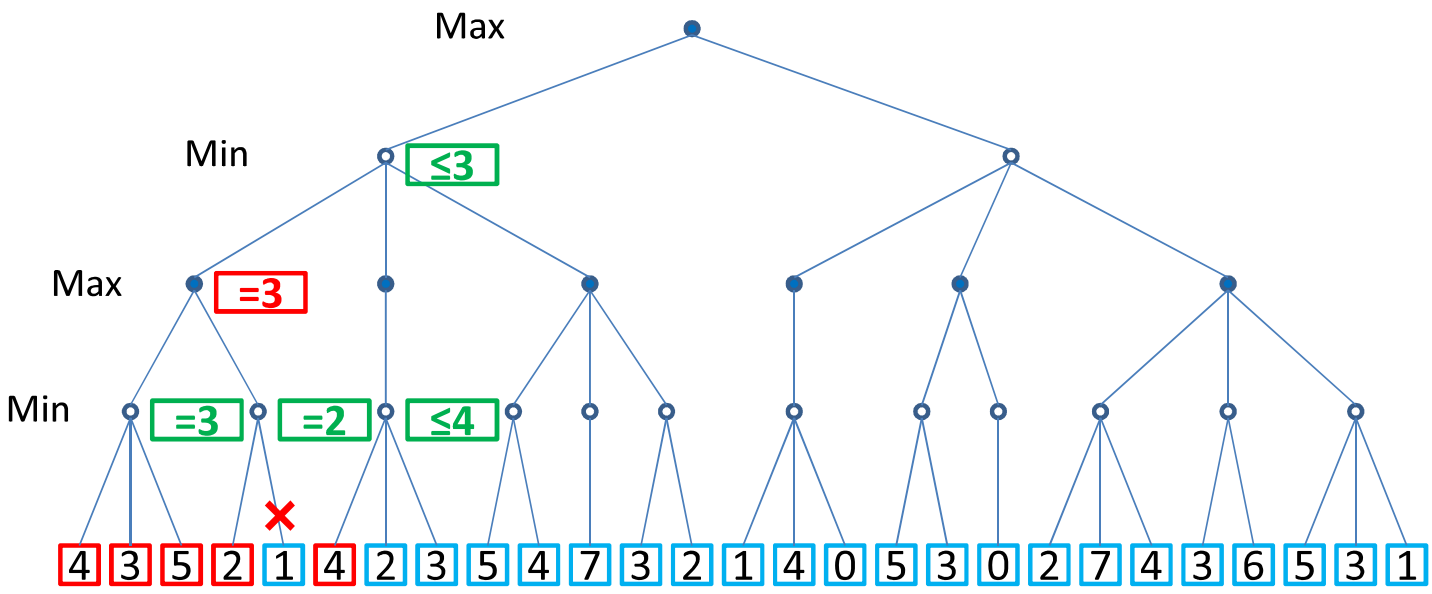
MiniMax with $\alpha\beta$-pruning



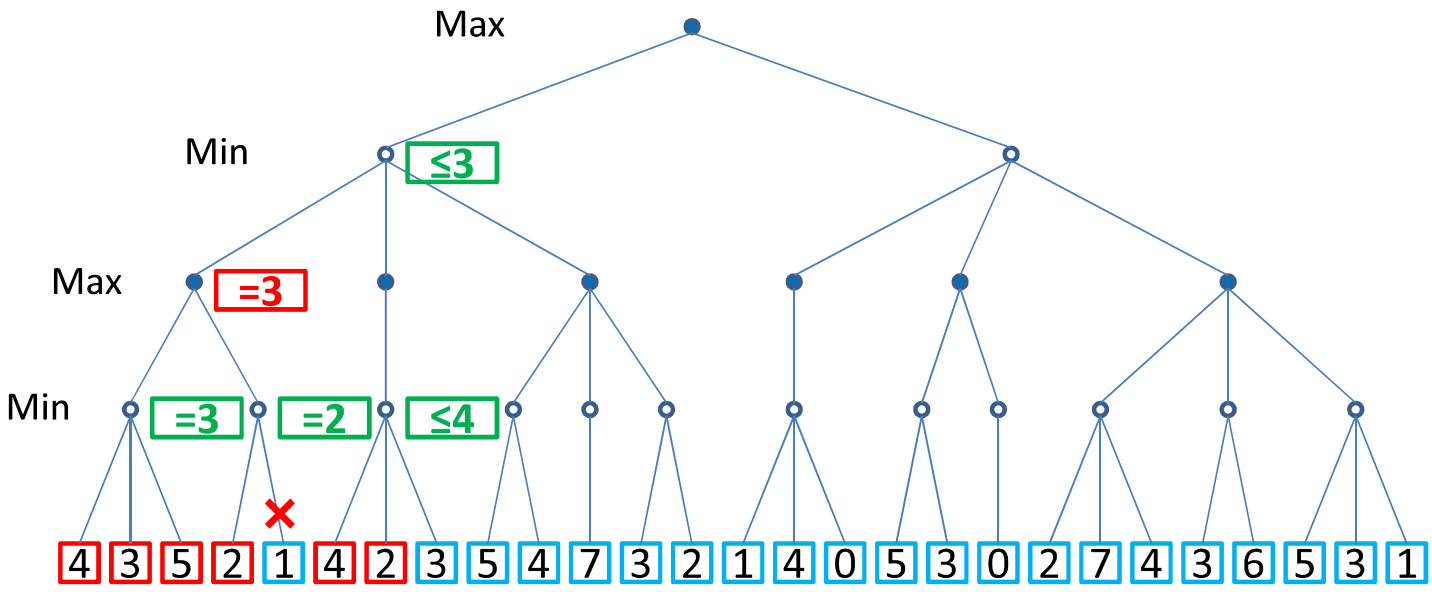
MiniMax with $\alpha\beta$-pruning



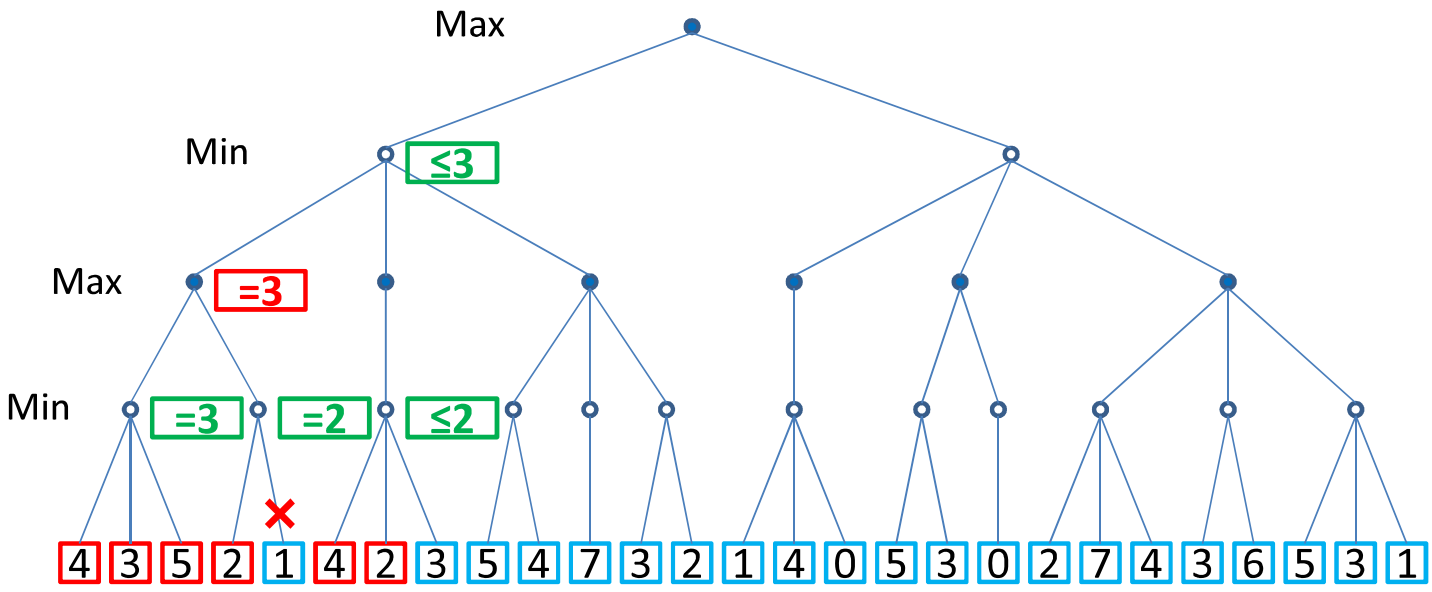
MiniMax with $\alpha\beta$-pruning



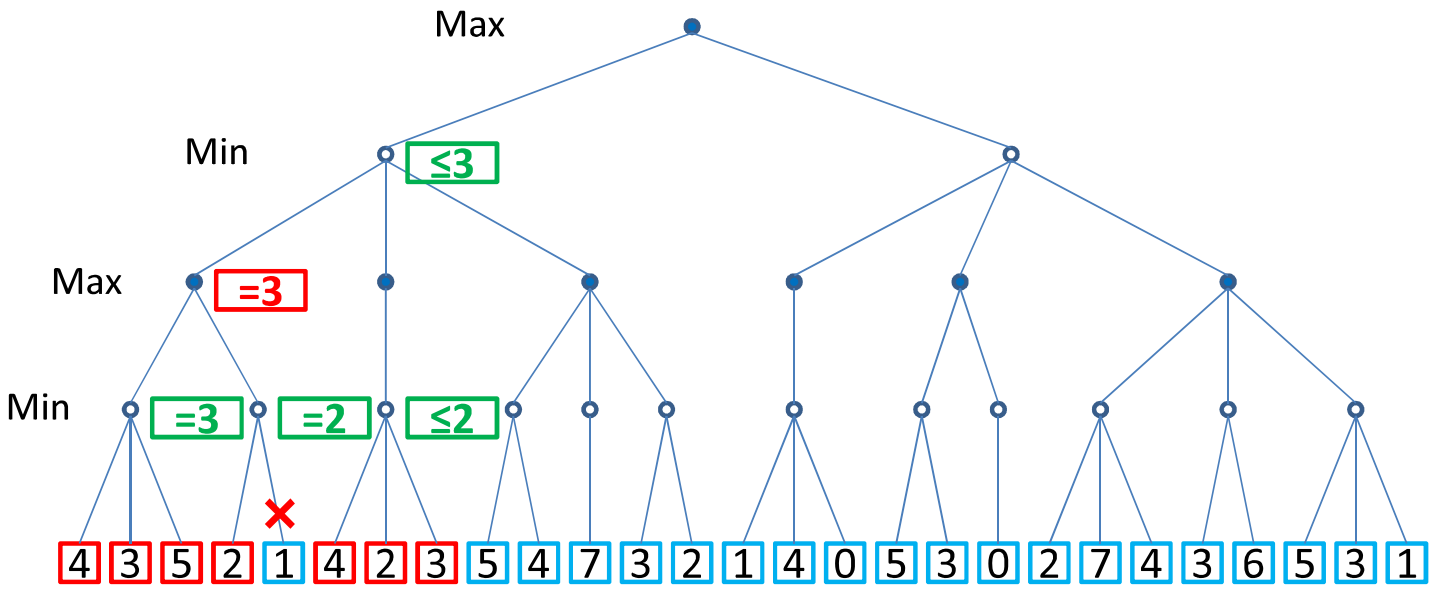
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning



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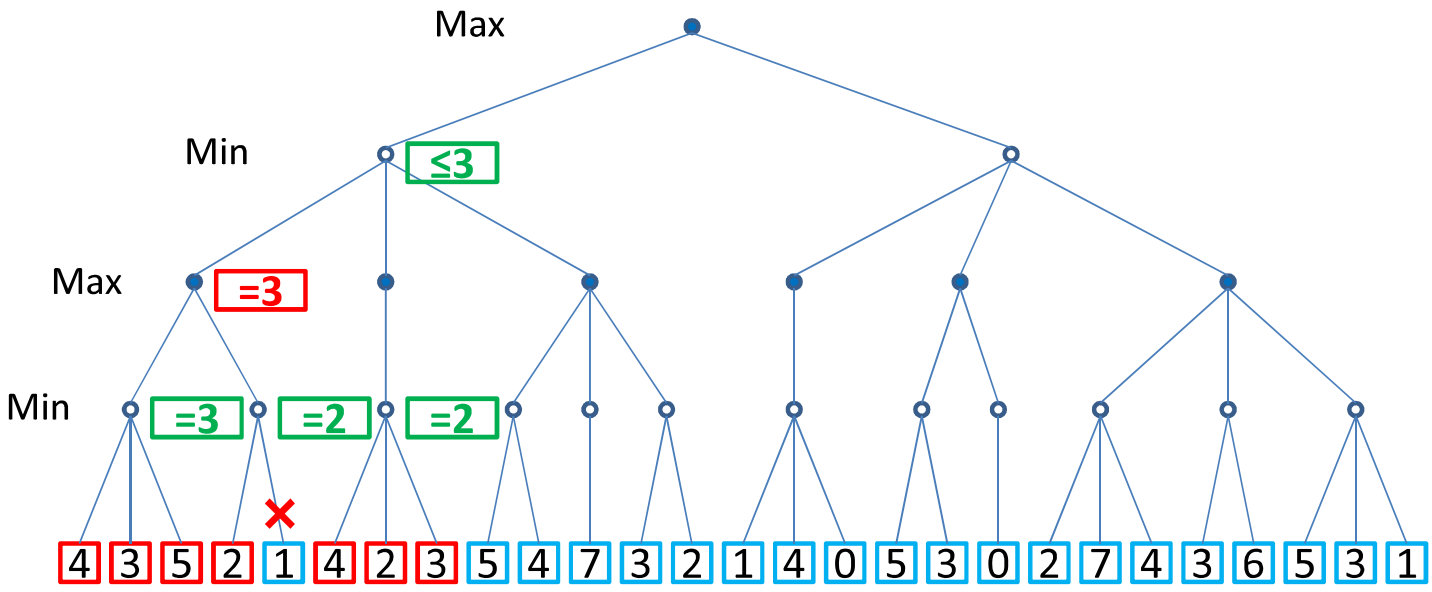

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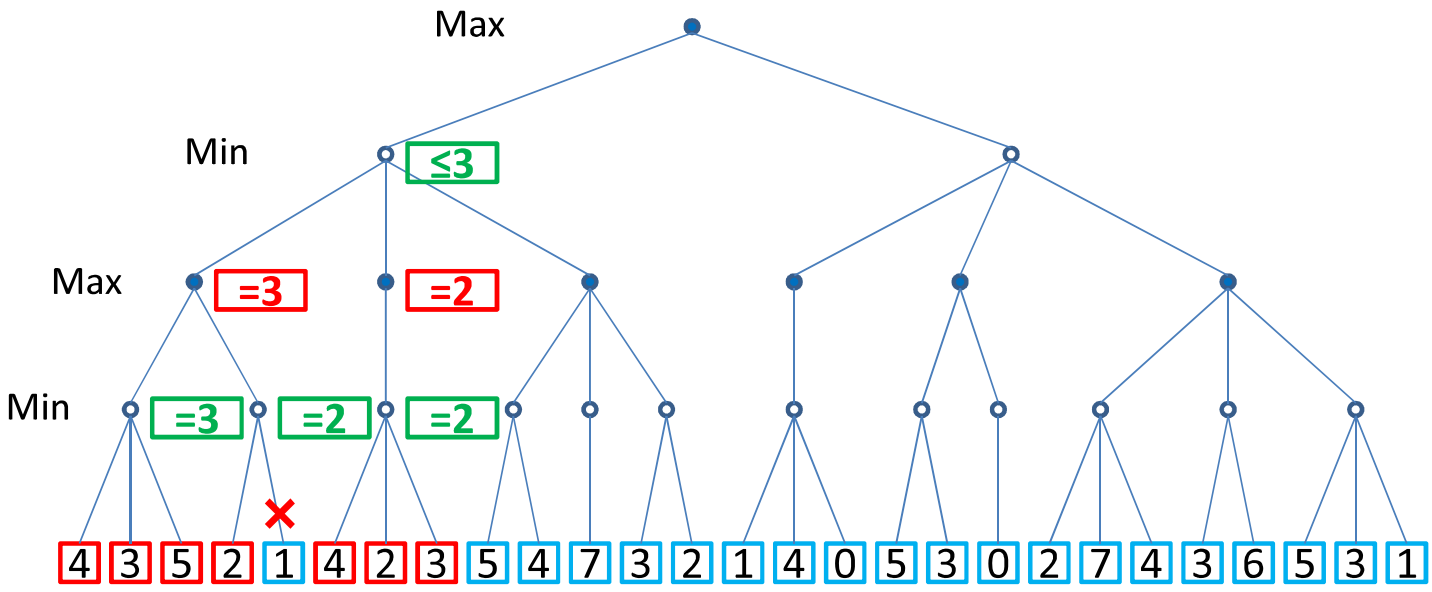

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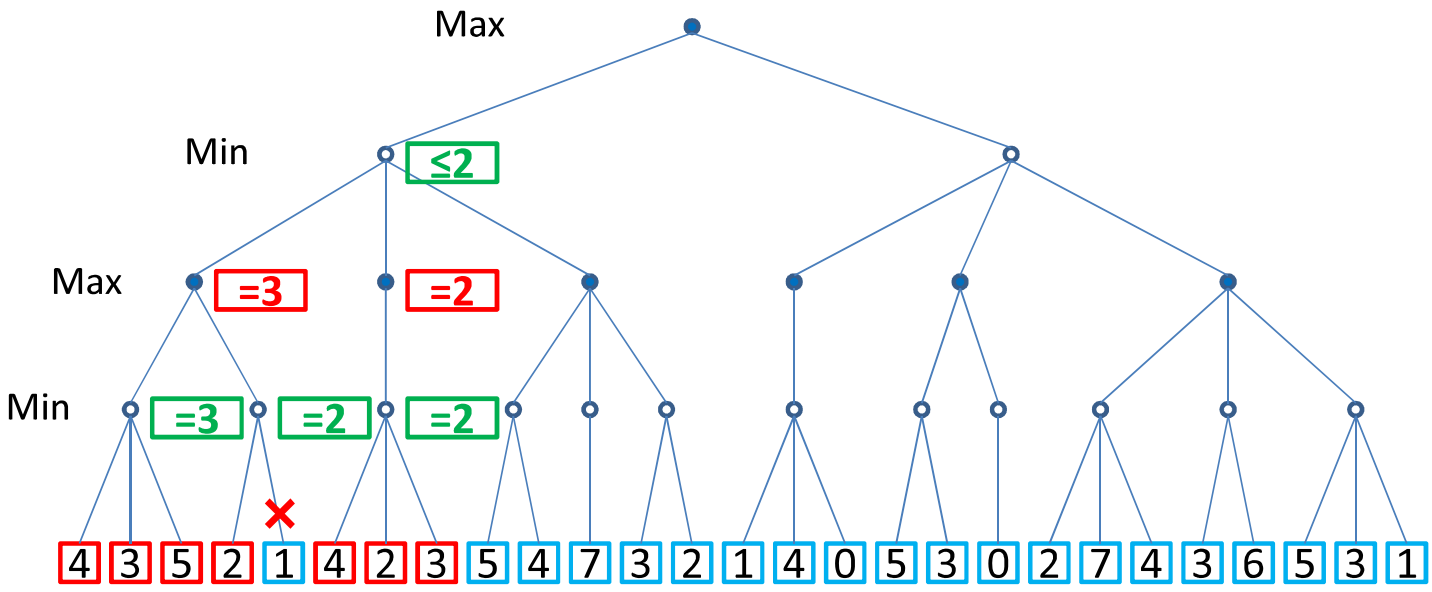
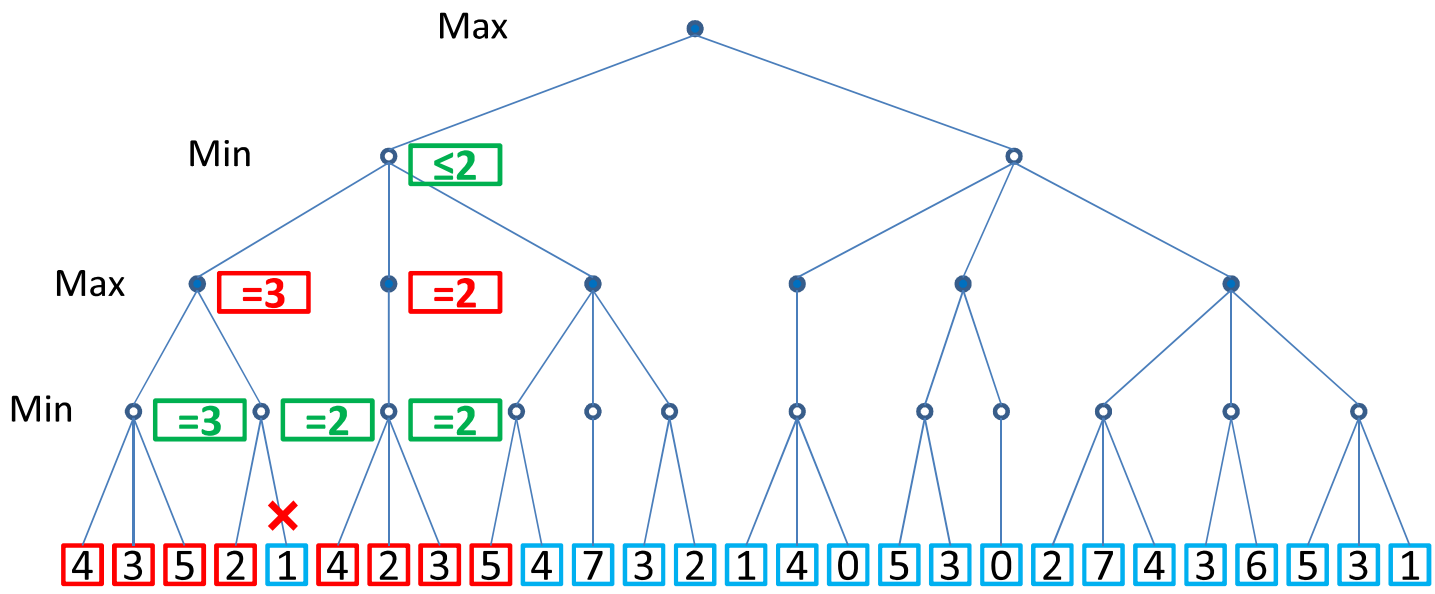
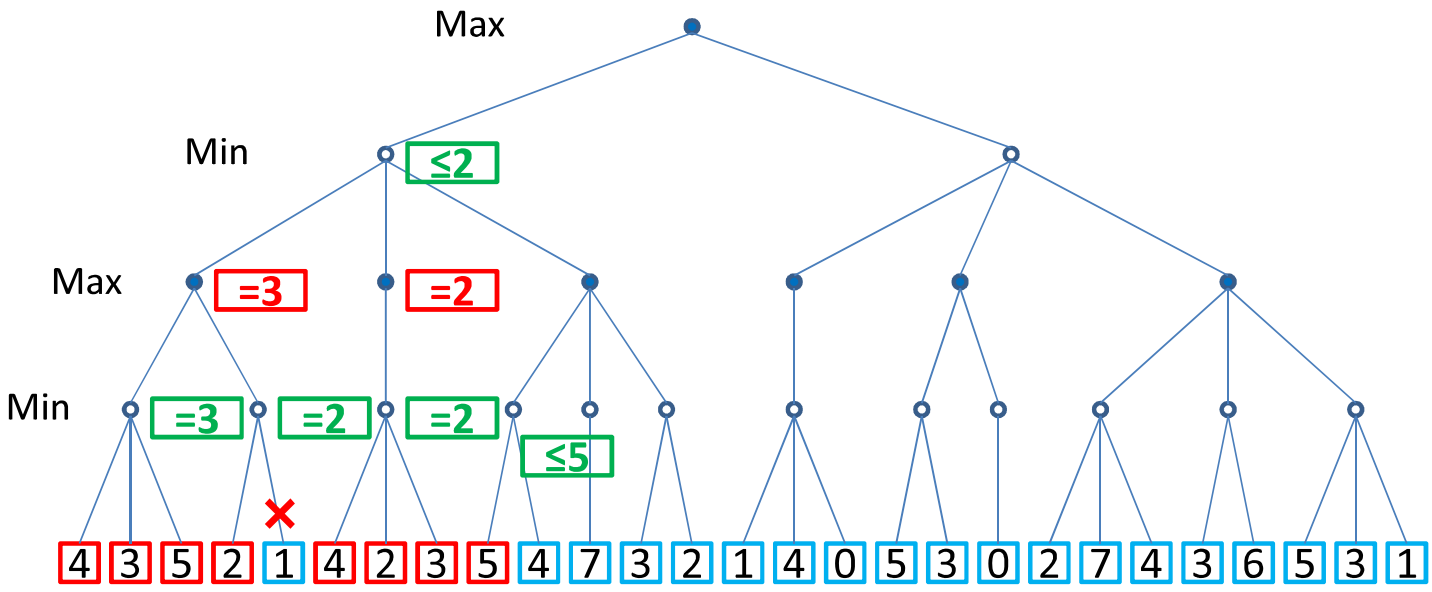


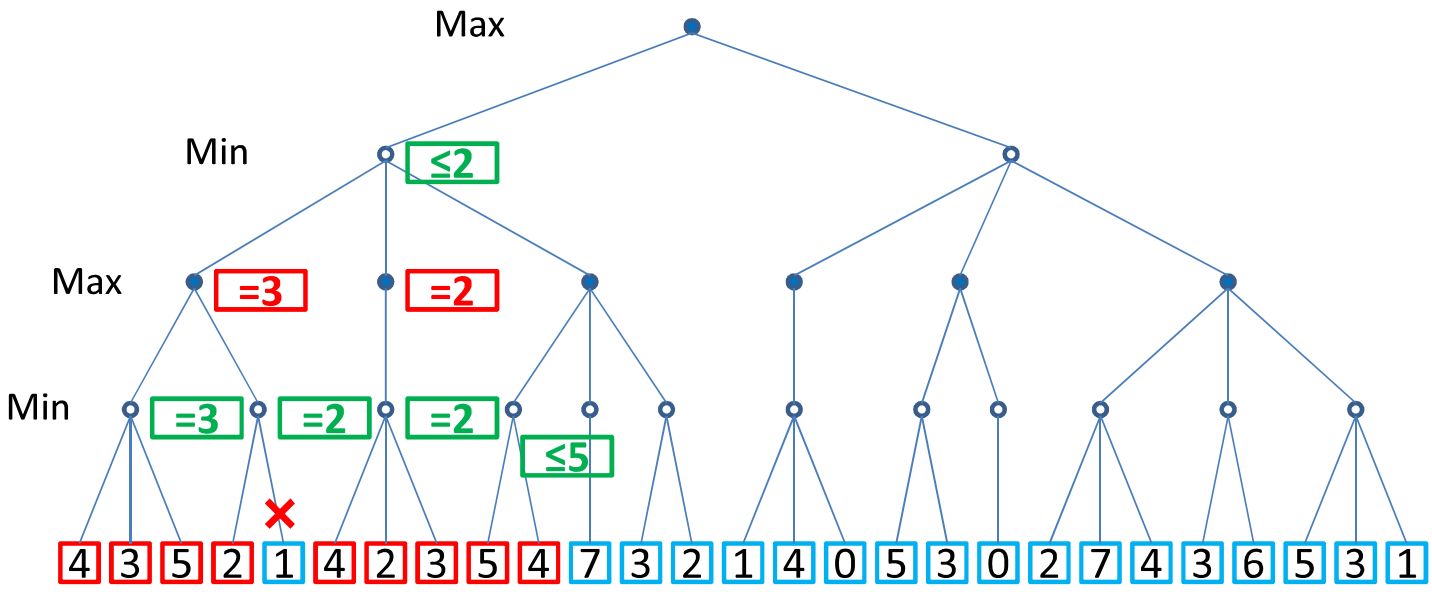
MiniMax with $\alpha\beta$-pruning



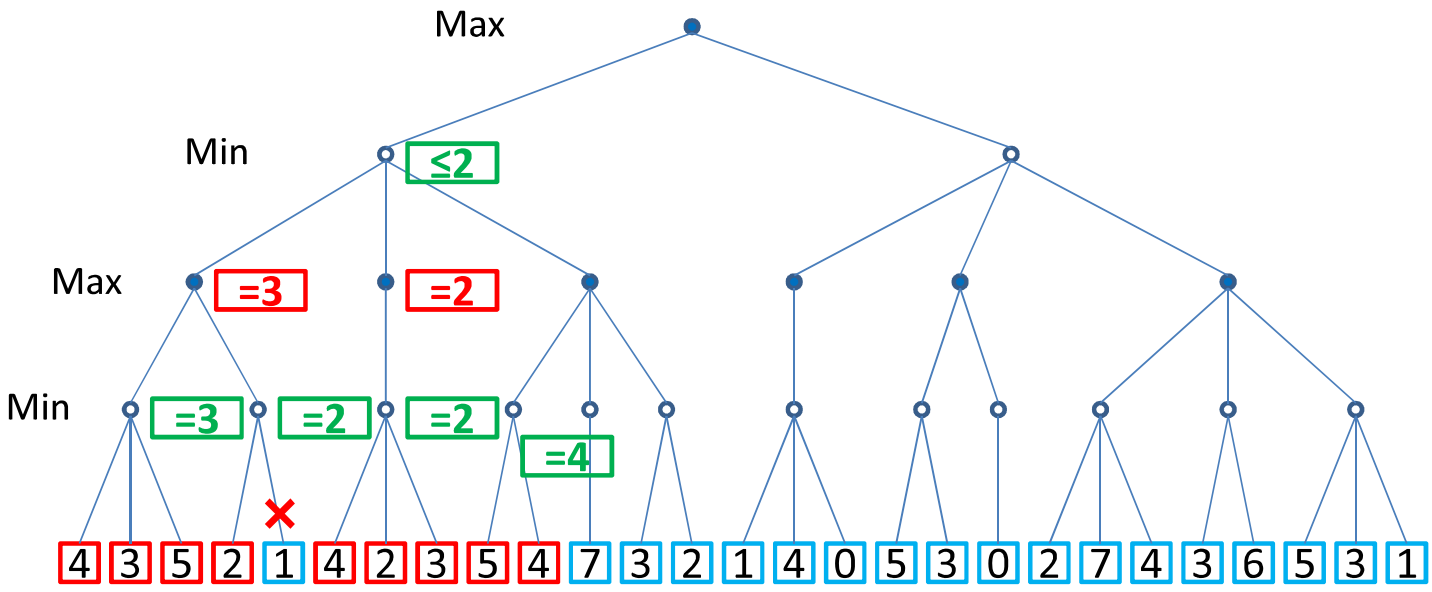
MiniMax with $\alpha\beta$-pruning



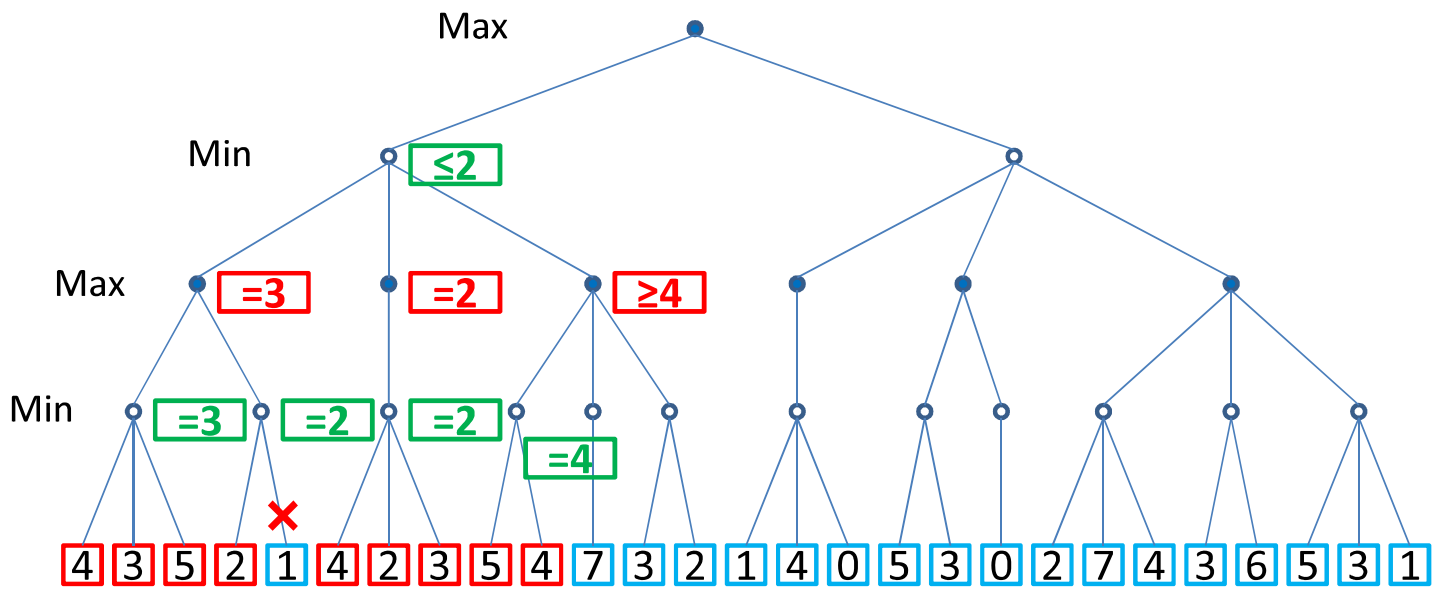
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning

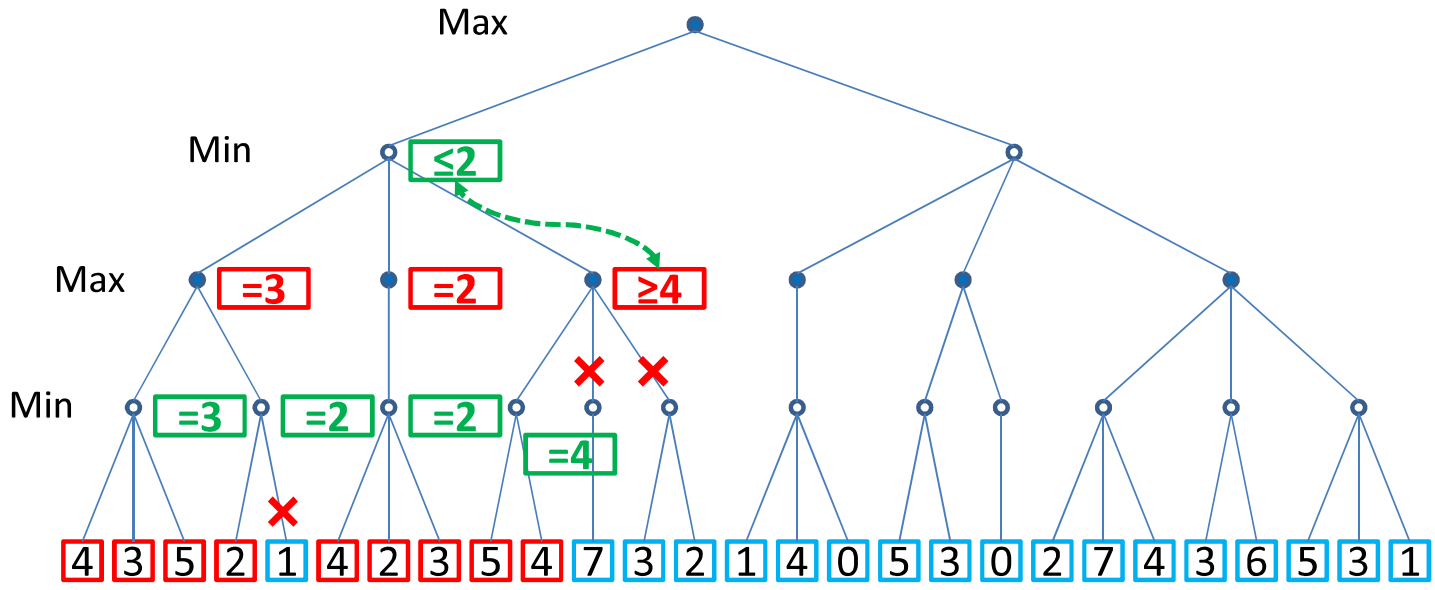


MiniMax with $\alpha\beta$-pruning

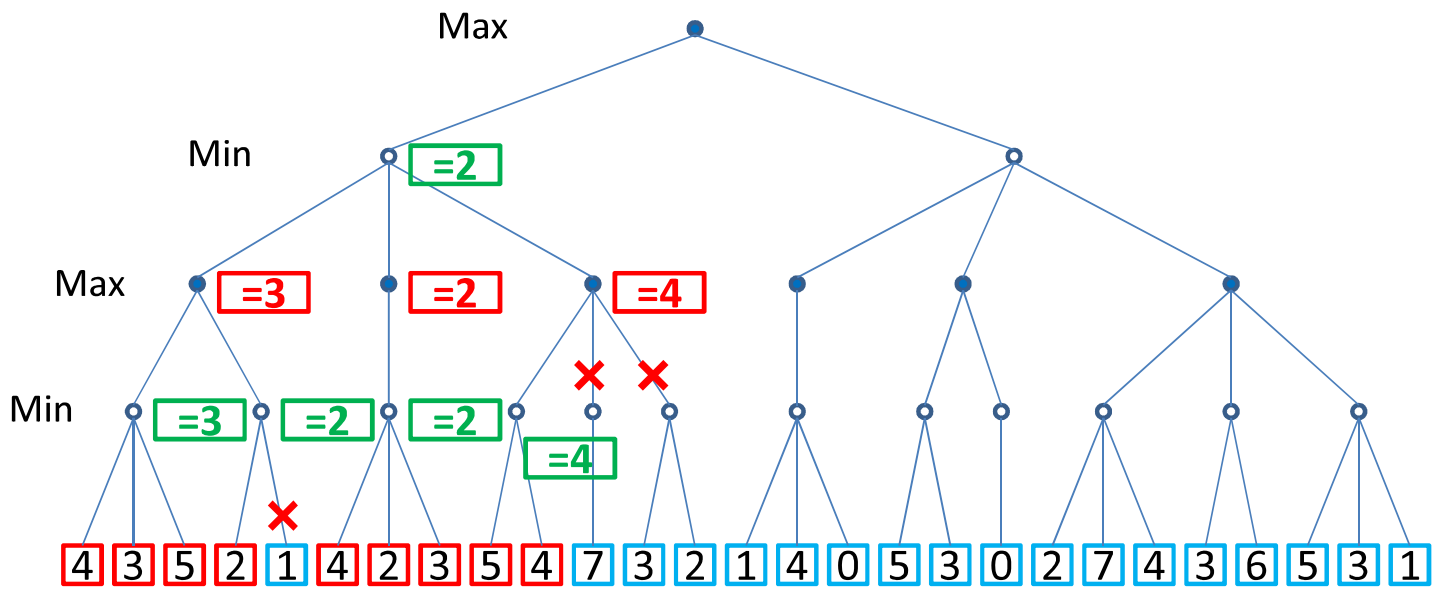


MiniMax with $\alpha\beta$-pruning

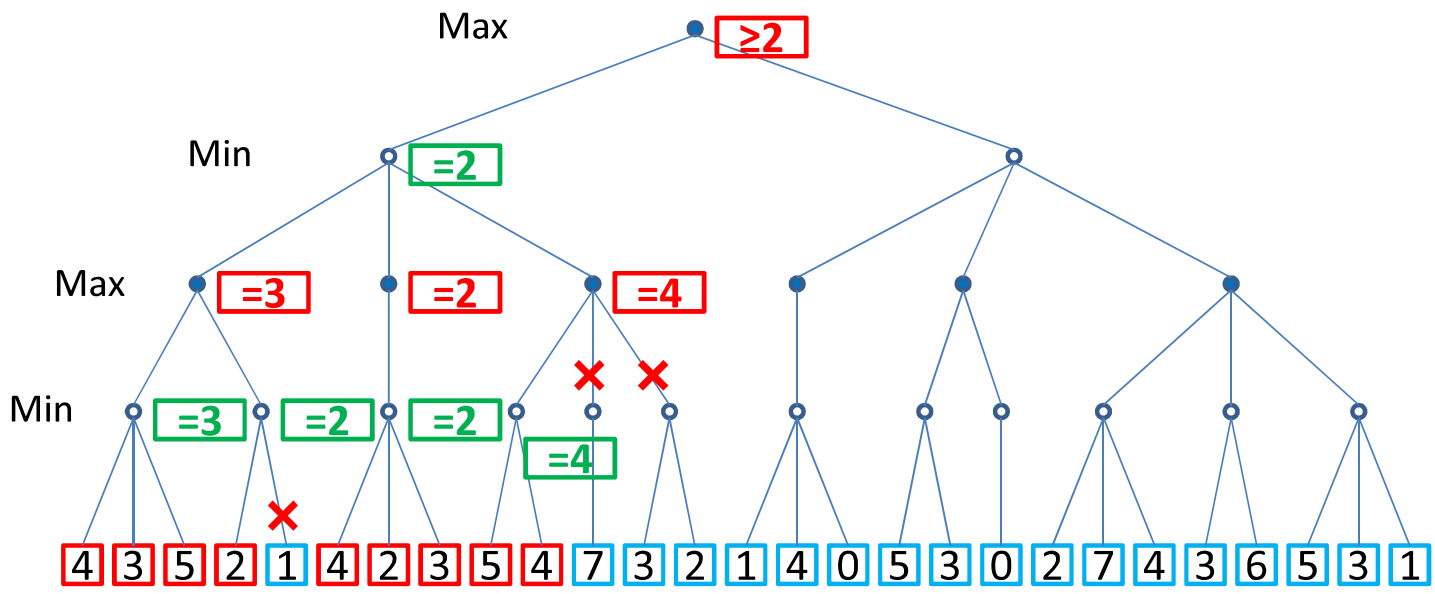
- **Prune: Parent α-node ≤ Child β-node**



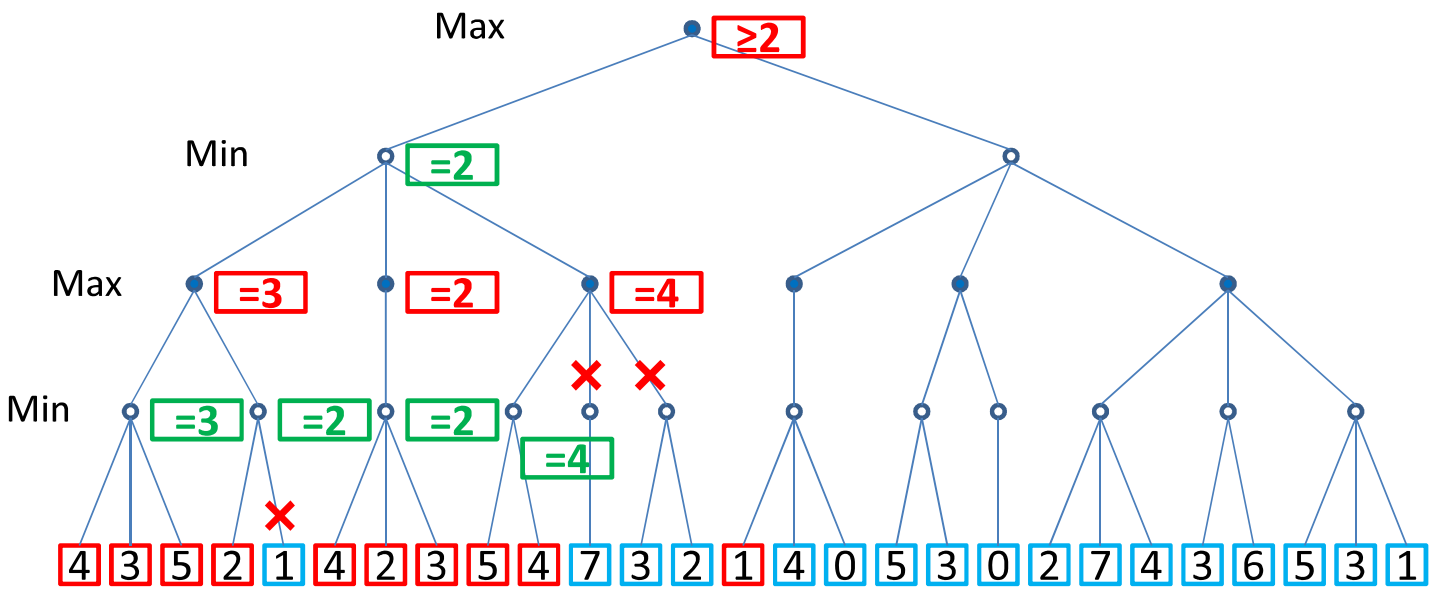
MiniMax with $\alpha\beta$-pruning



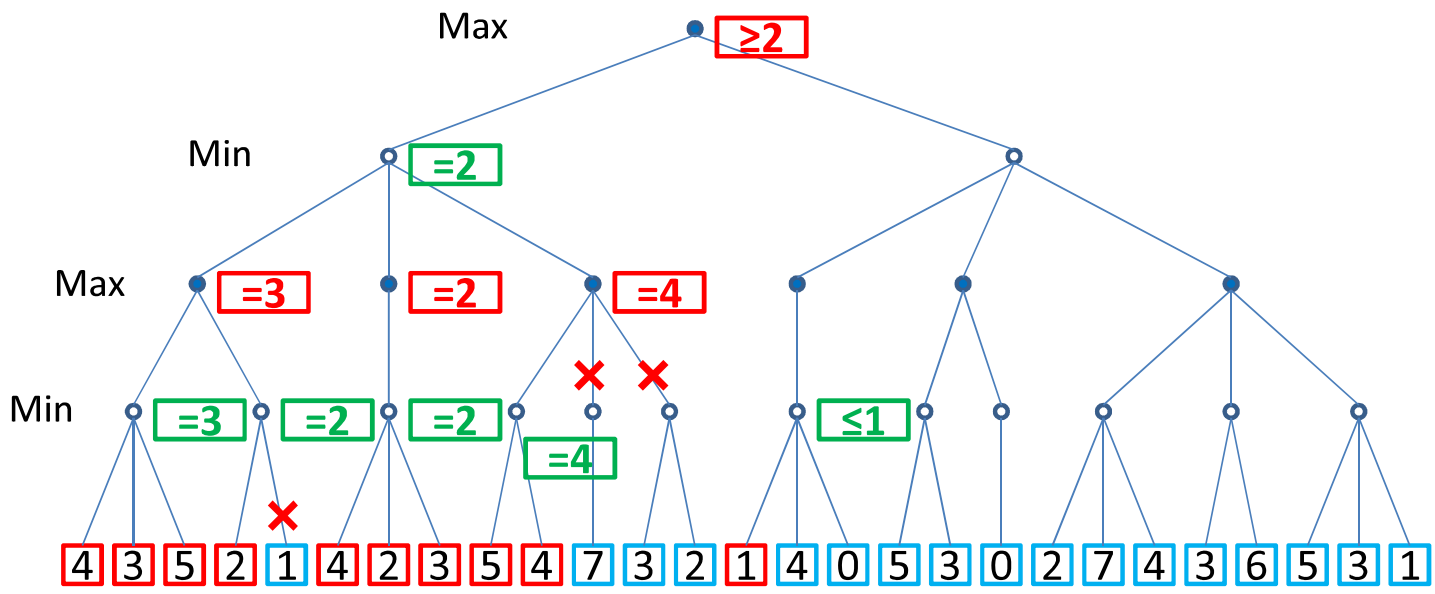
MiniMax with $\alpha\beta$-pruning



MiniMax with $\alpha\beta$-pruning

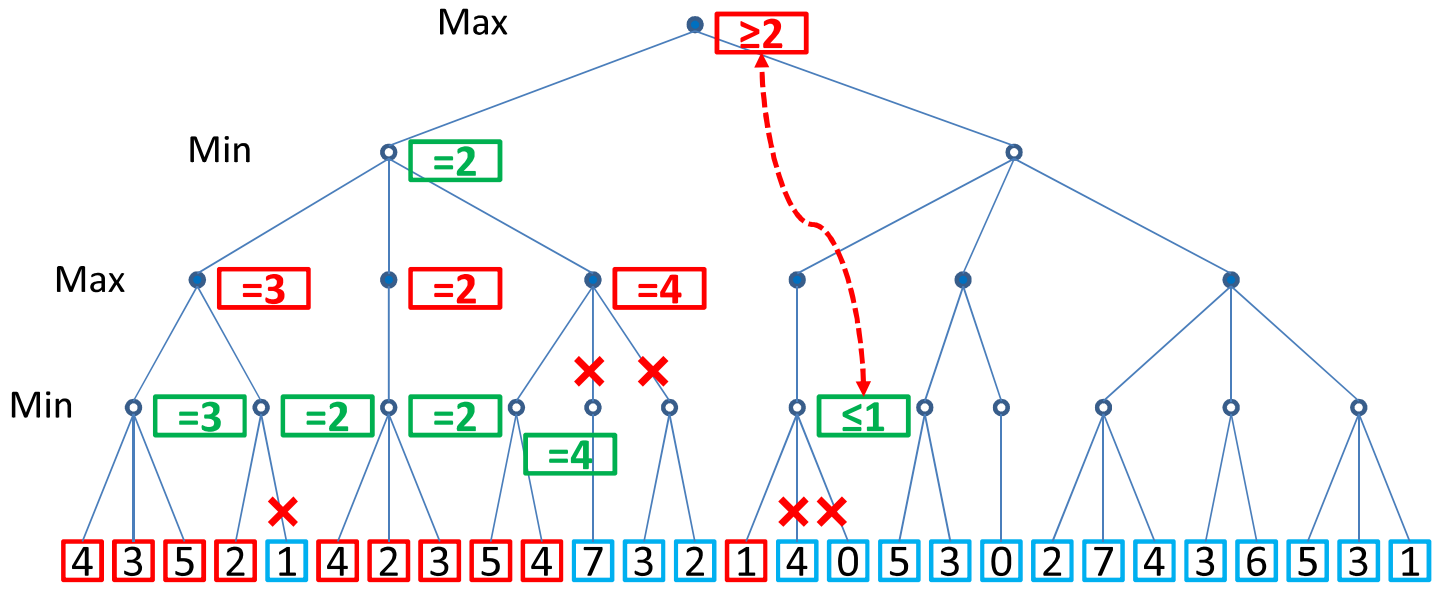


MiniMax with $\alpha\beta$-pruning

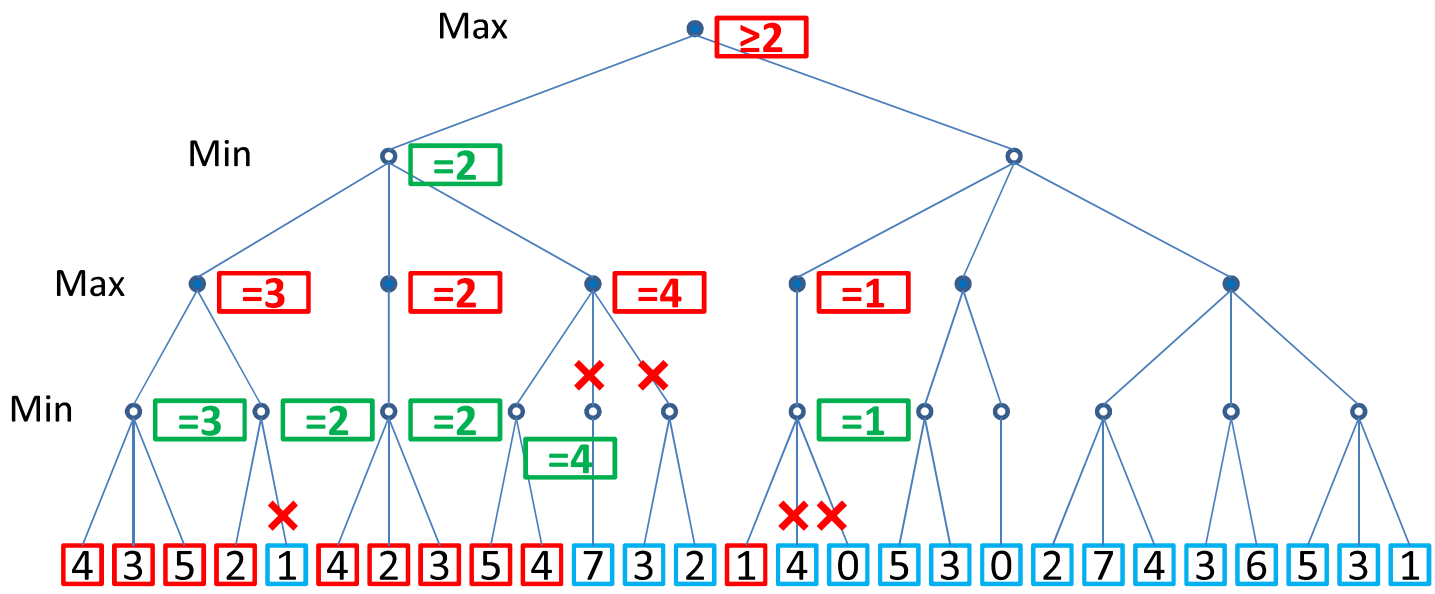


MiniMax with $\alpha\beta$-pruning

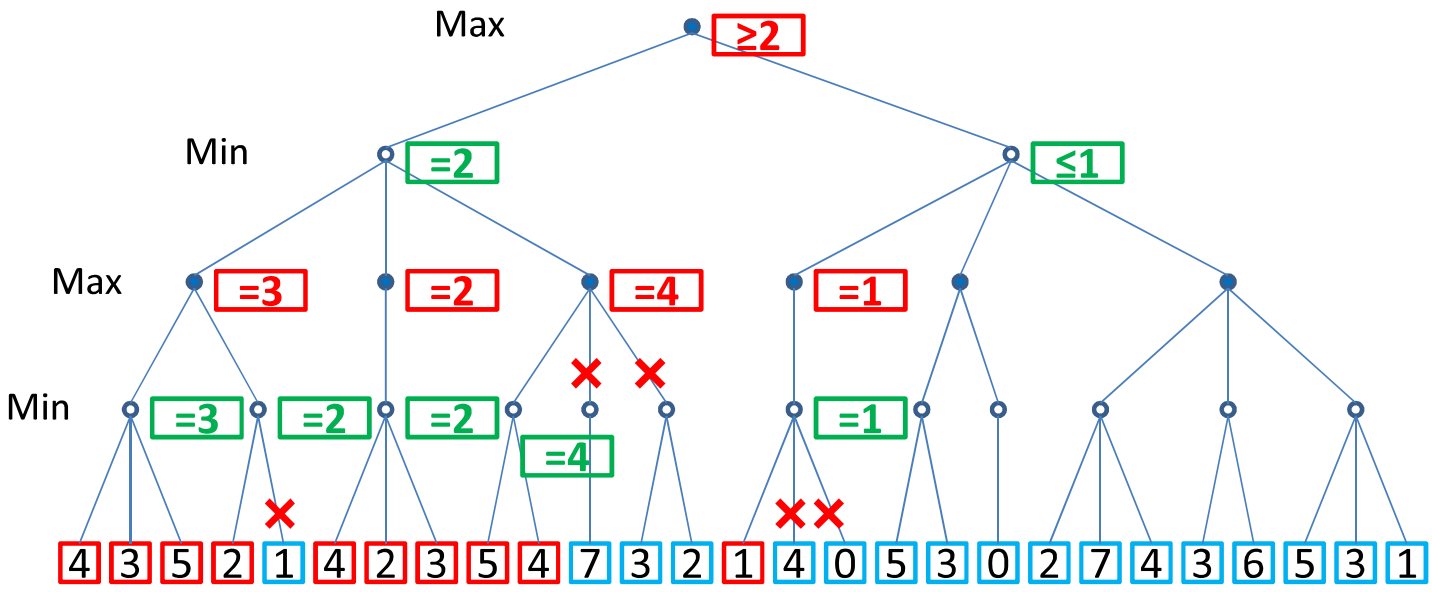
- **"Deep" cut-off: Ancestor β-node ≥ α-node**



MiniMax with $\alpha\beta$-pruning

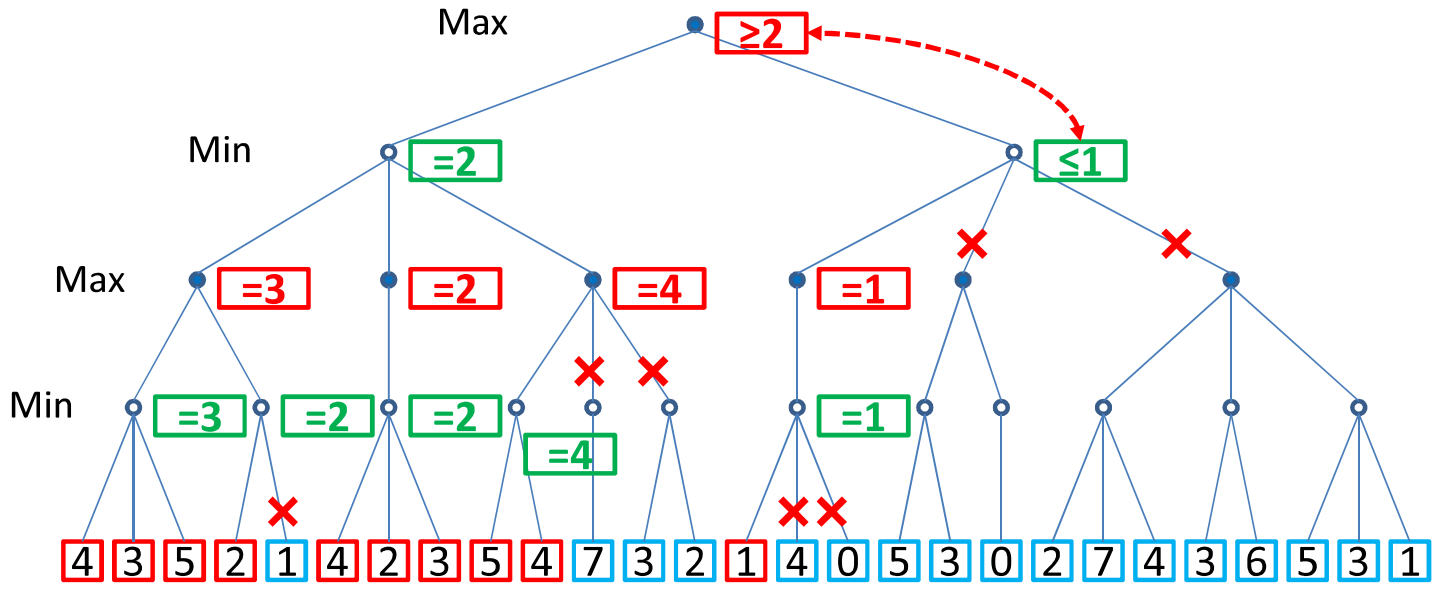


MiniMax with $\alpha\beta$-pruning



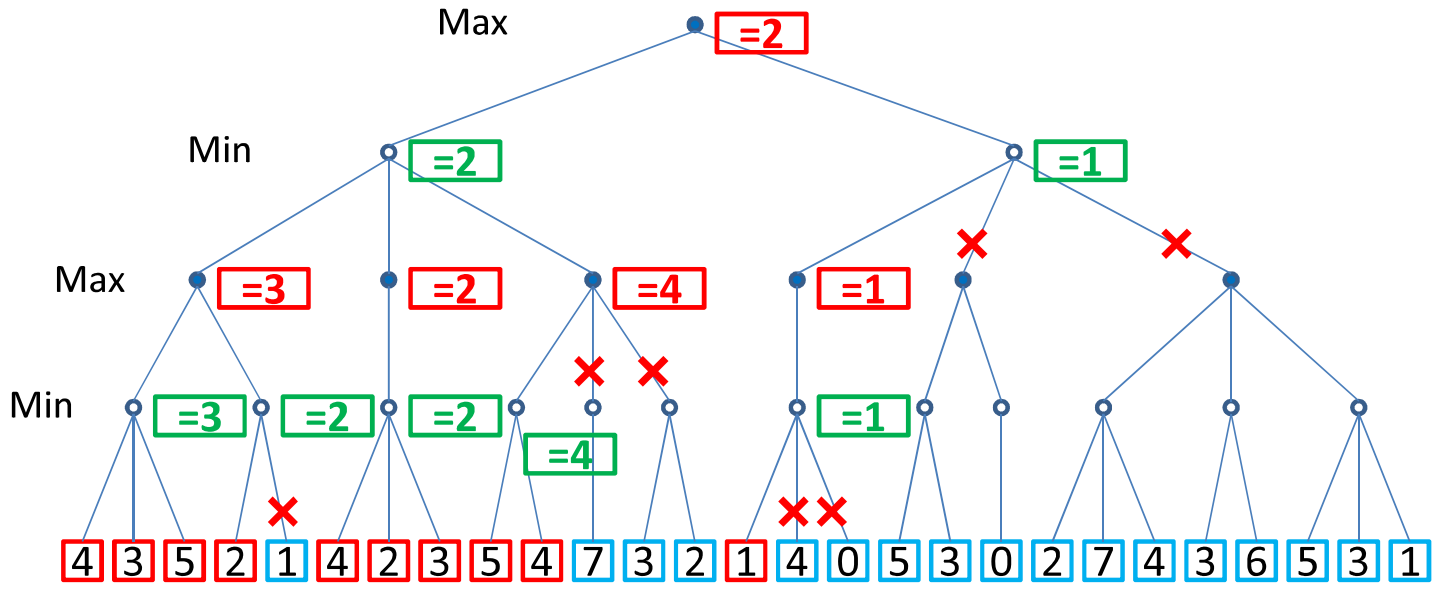
MiniMax with $\alpha\beta$-pruning

- **Prune: Parent β-node ≥ Child α-node**



MiniMax with $\alpha\beta$-pruning

- **17 static evaluations saved**

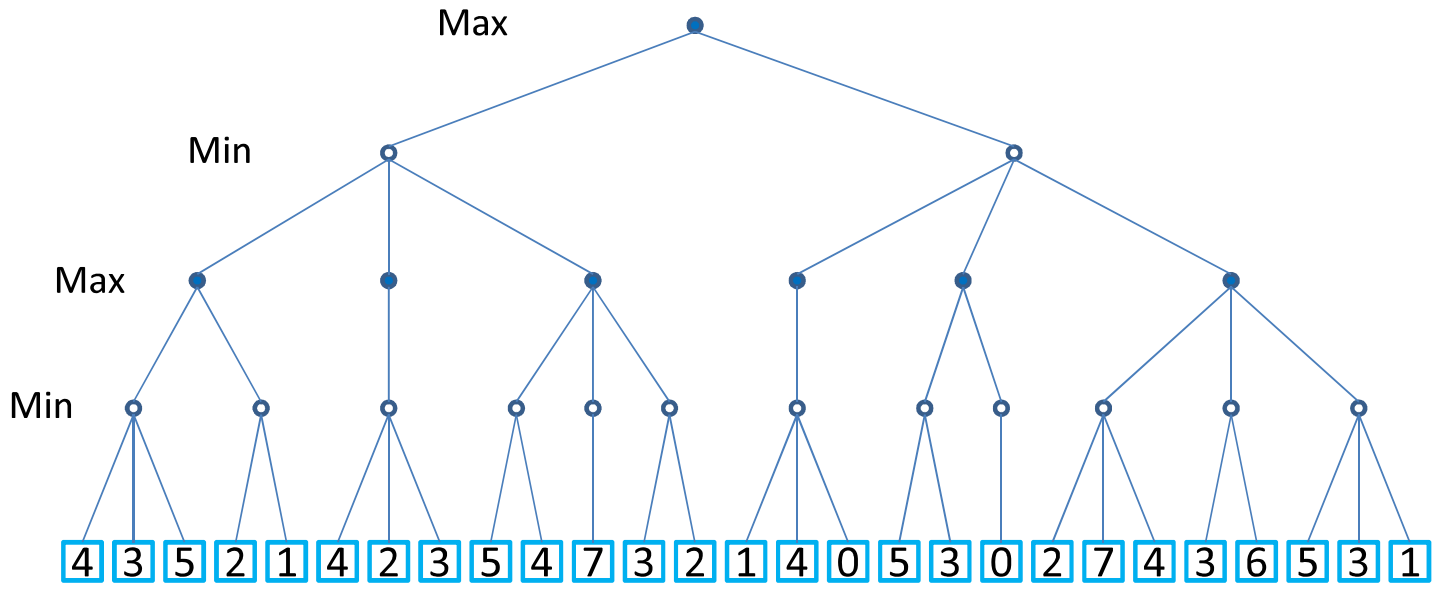


MiniMax & Constraint Processing: MiniMax Algorithm

PROBLEM 2

Problem 2

- Can the nodes be ordered in such a way that $\alpha\beta$-pruning can cut off more branches?

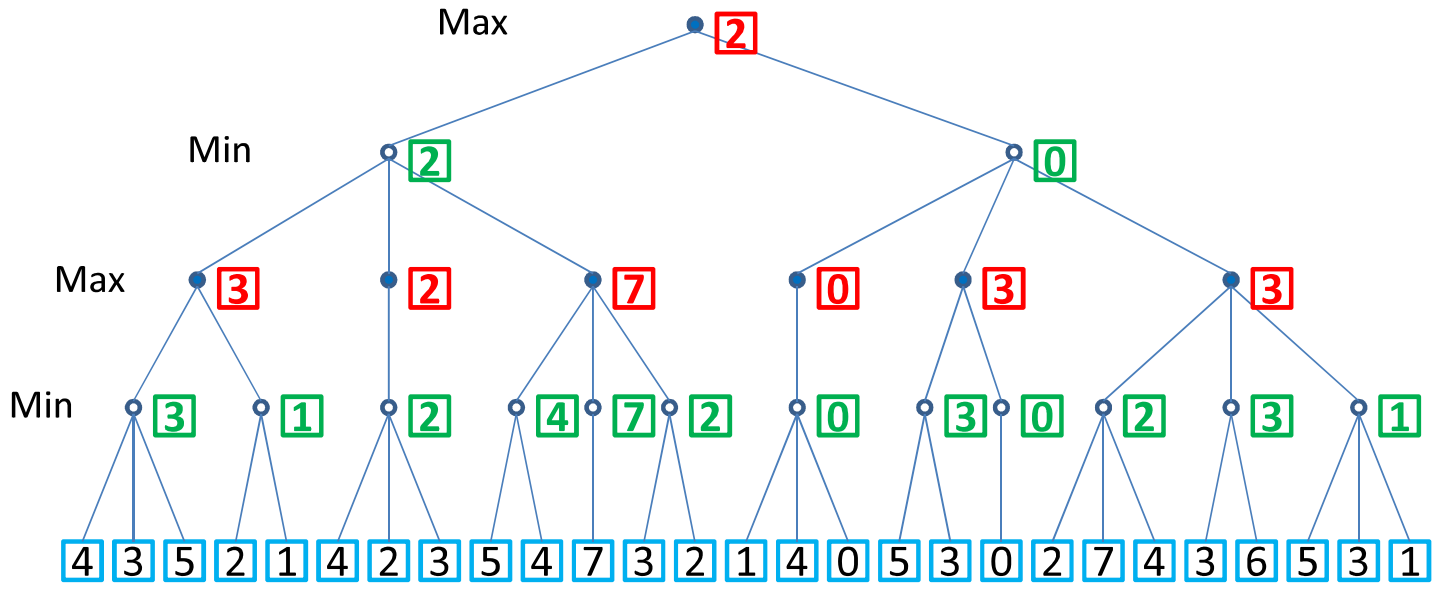


MiniMax & Constraint Processing: MiniMax Algorithm

OPTIMIZING $\alpha\beta$-PRUNING

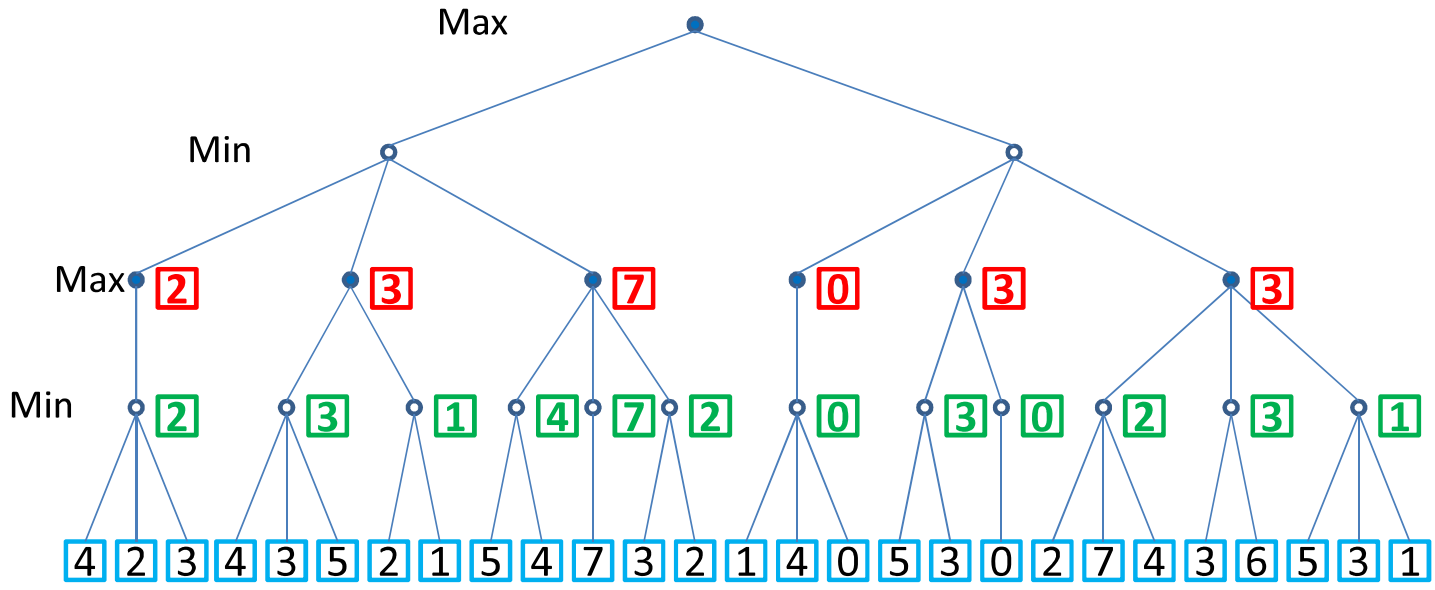
Optimizing $\alpha\beta$-Pruning

- **Best case:** Each layer best node left-to-right



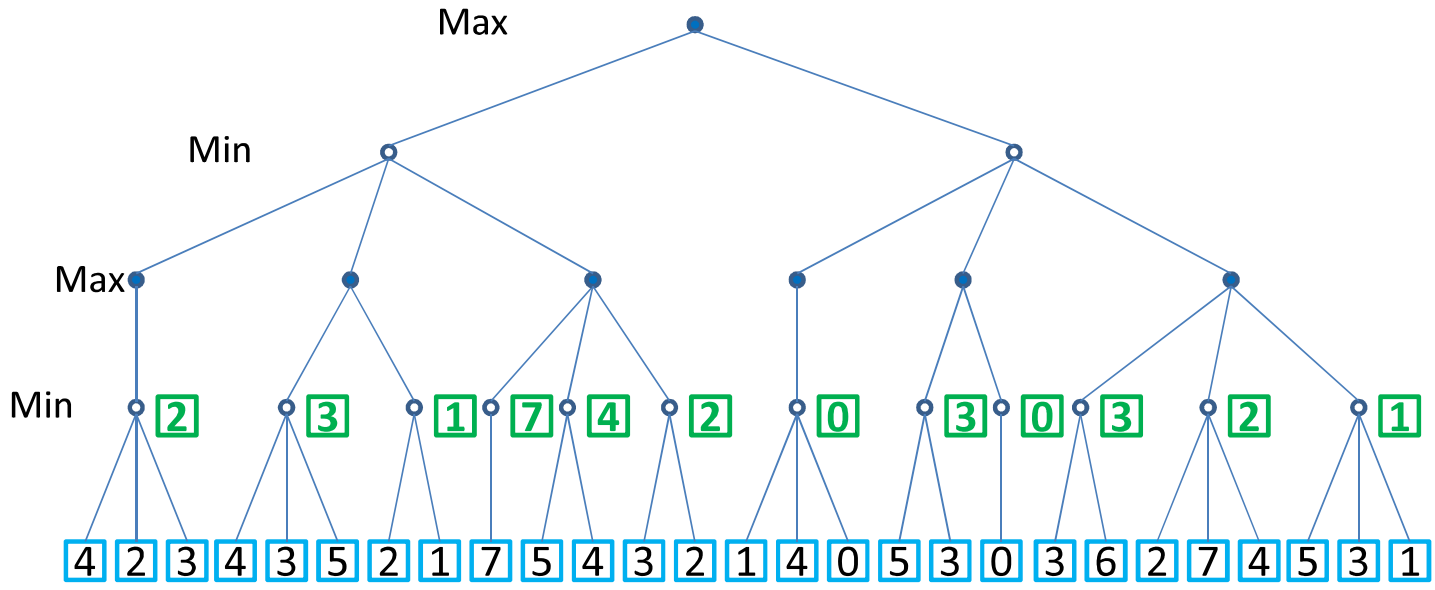
Optimizing $\alpha\beta$-Pruning

- **Best case:** Each layer best node left-to-right



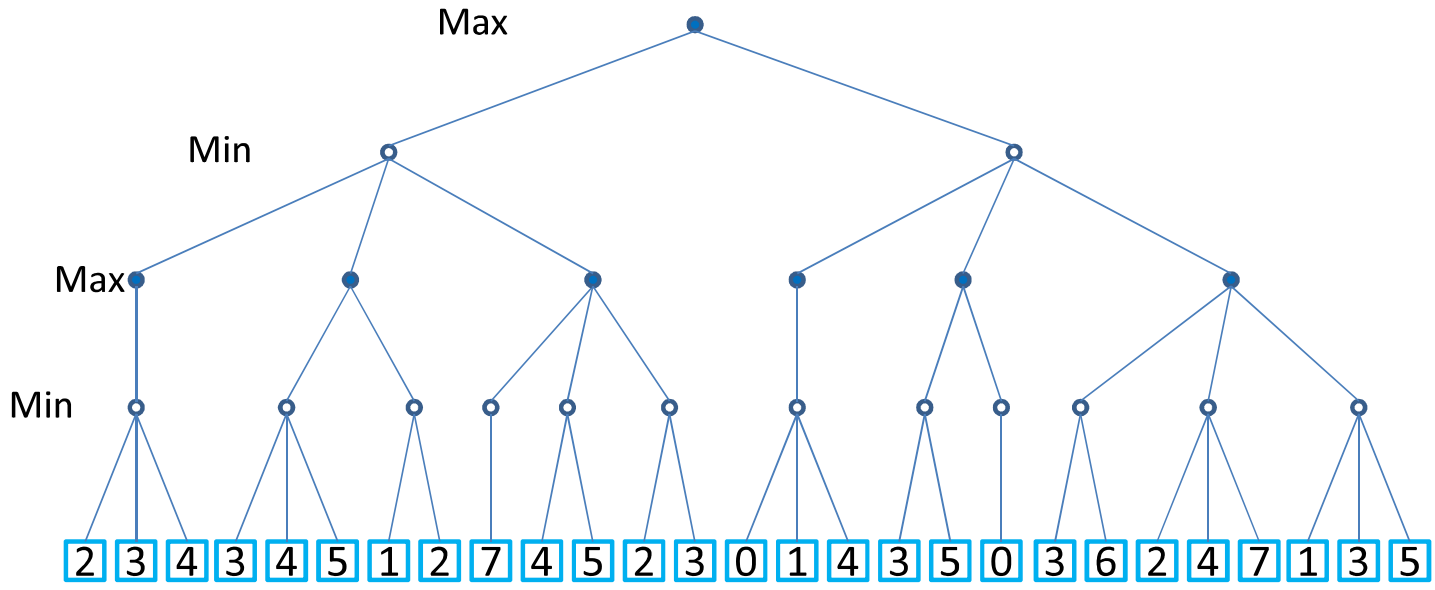
Optimizing $\alpha\beta$-Pruning

- **Best case:** Each layer best node left-to-right



Optimizing $\alpha\beta$-Pruning

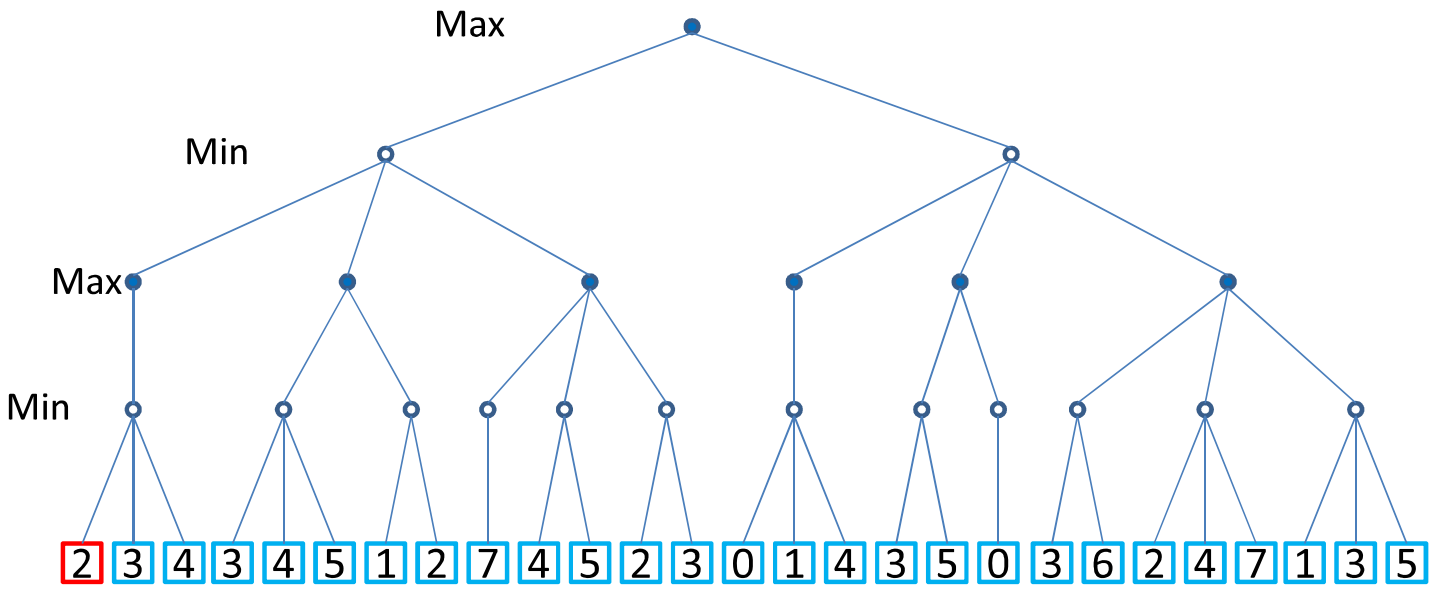
- **Best case:** Each layer best node left-to-right



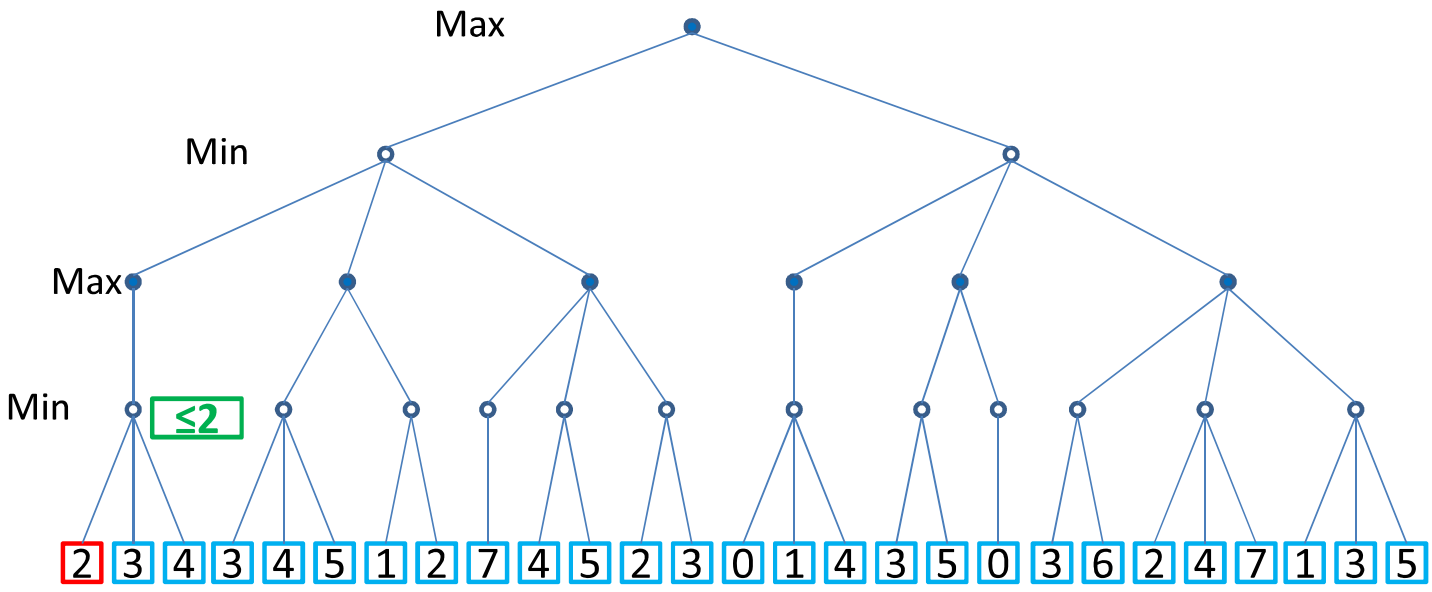
MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX WITH $\alpha\beta$-PRUNING

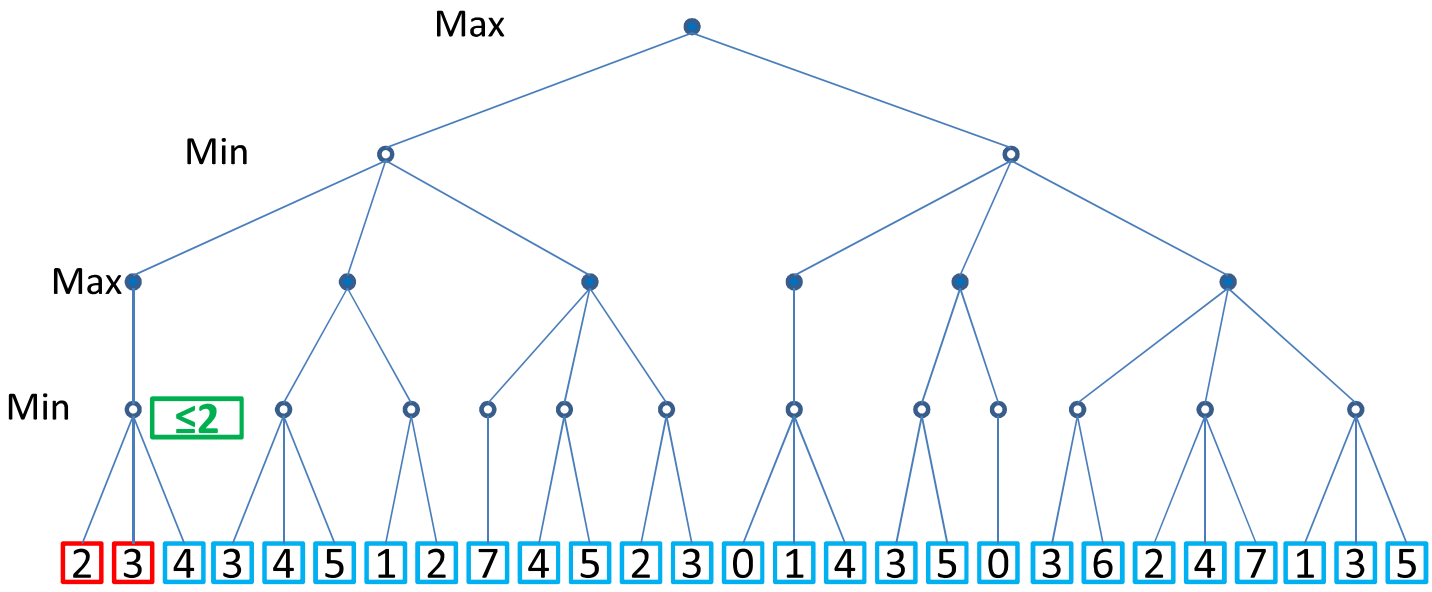
Minimax with $\alpha\beta$-Pruning



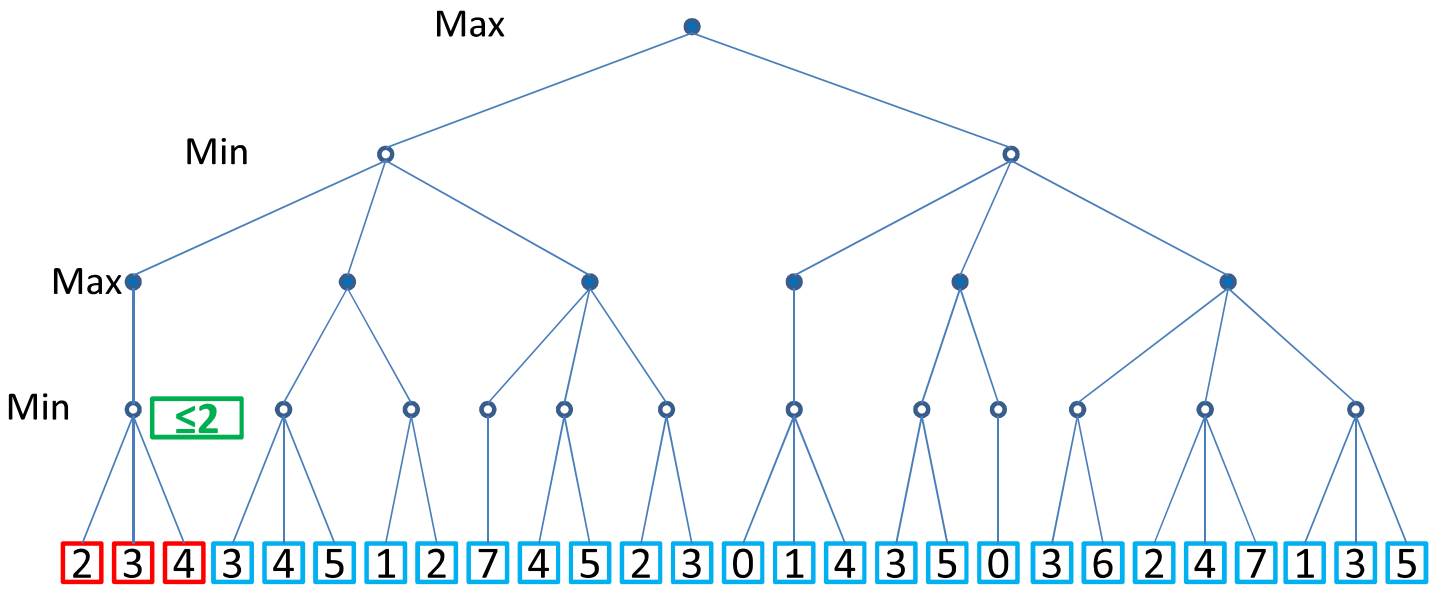
Minimax with $\alpha\beta$-Pruning



Minimax with $\alpha\beta$-Pruning



Minimax with $\alpha\beta$-Pruning



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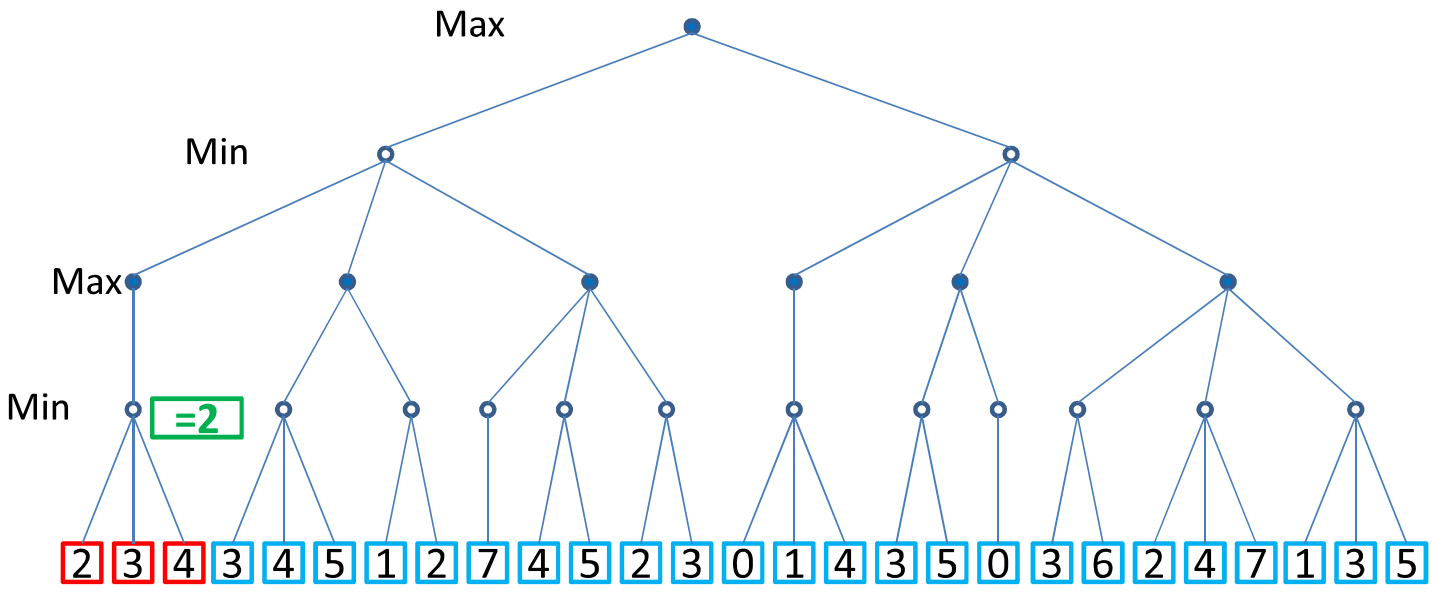

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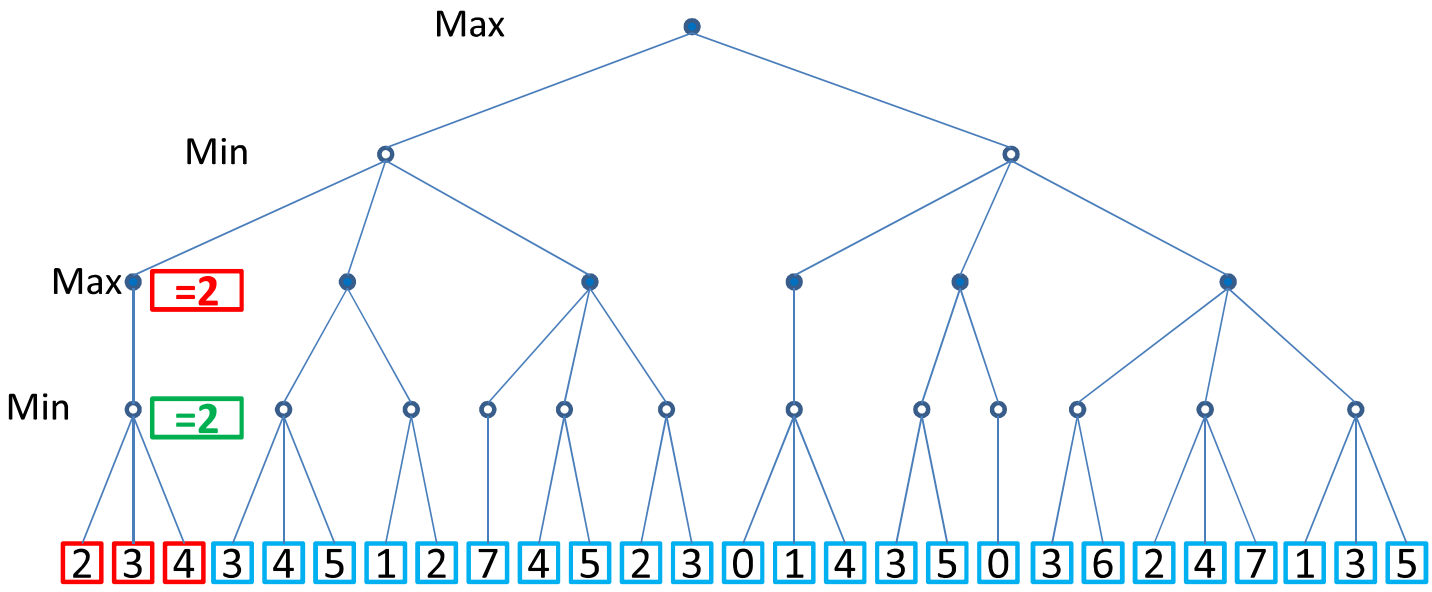

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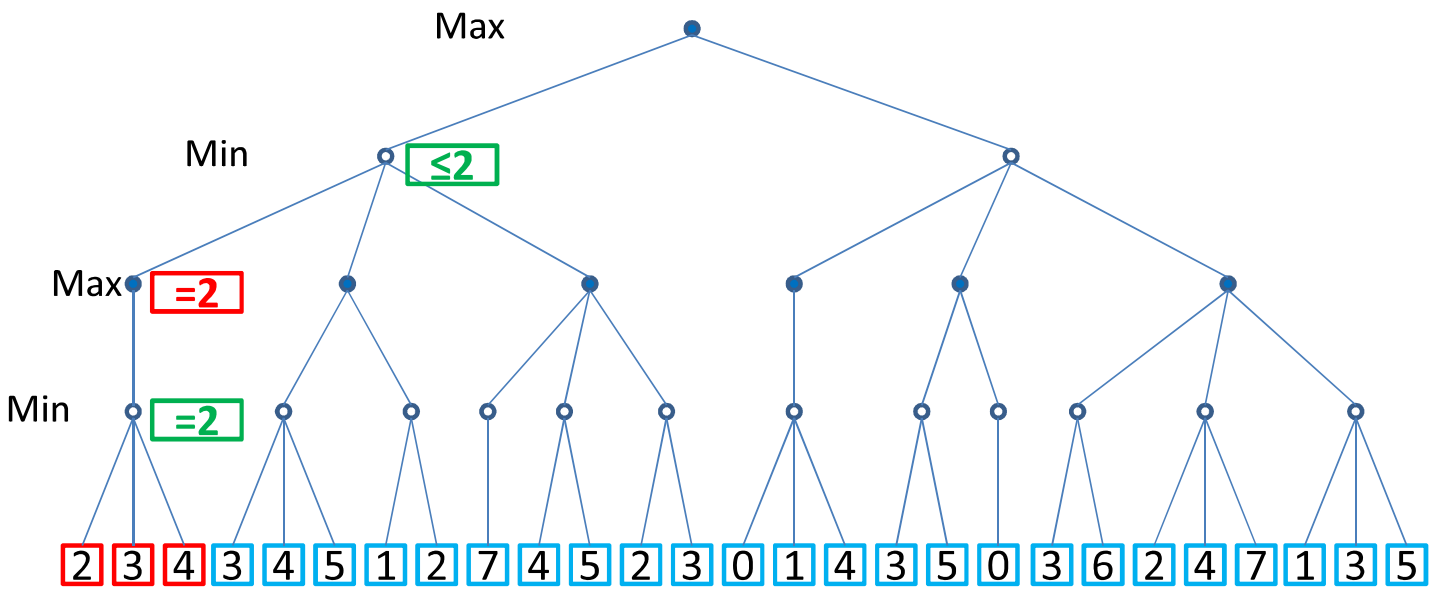

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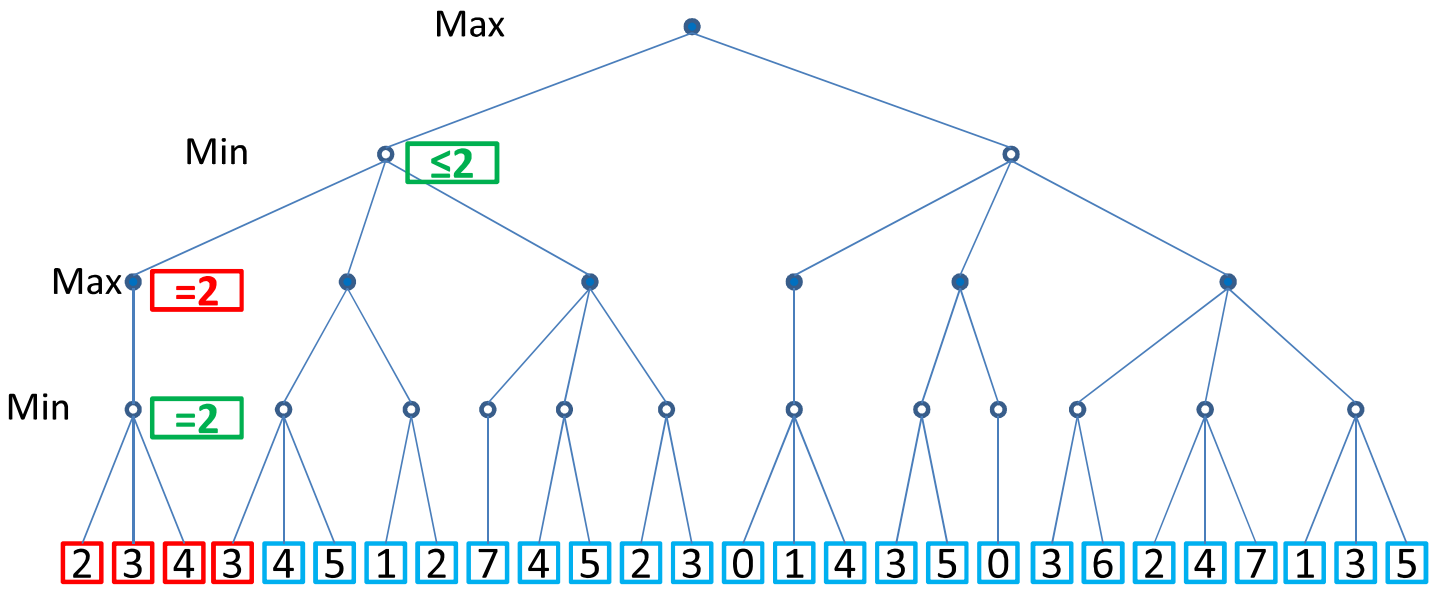

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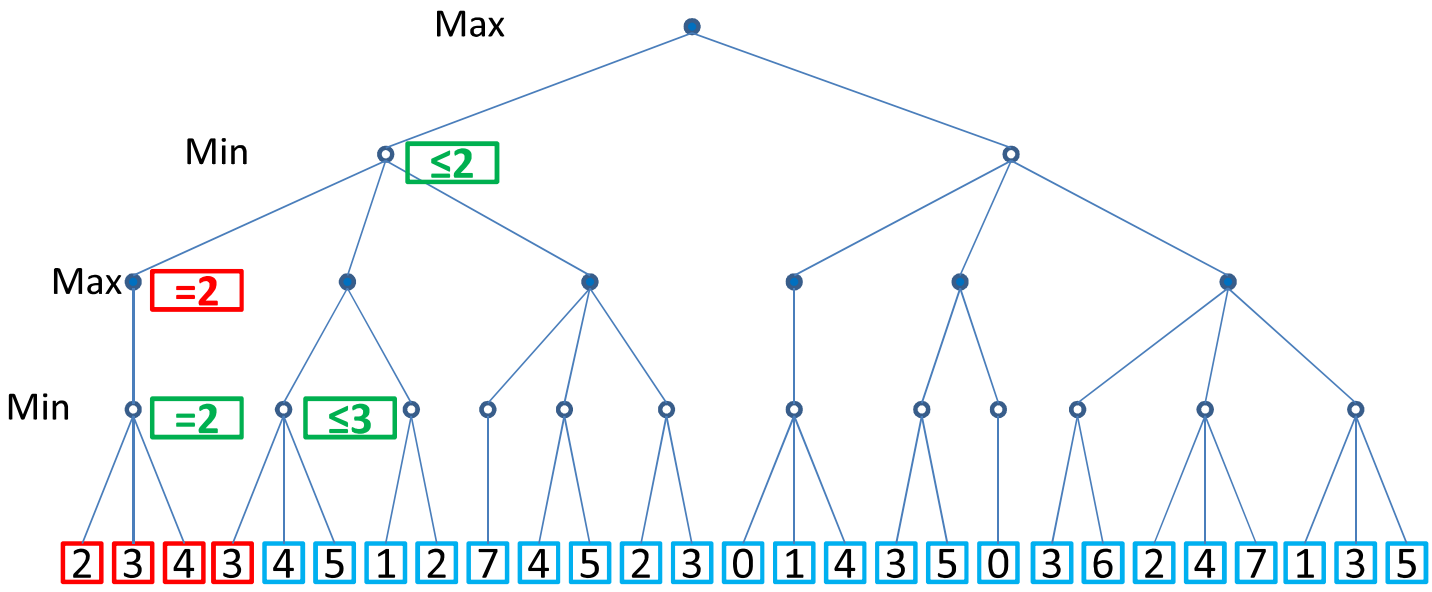

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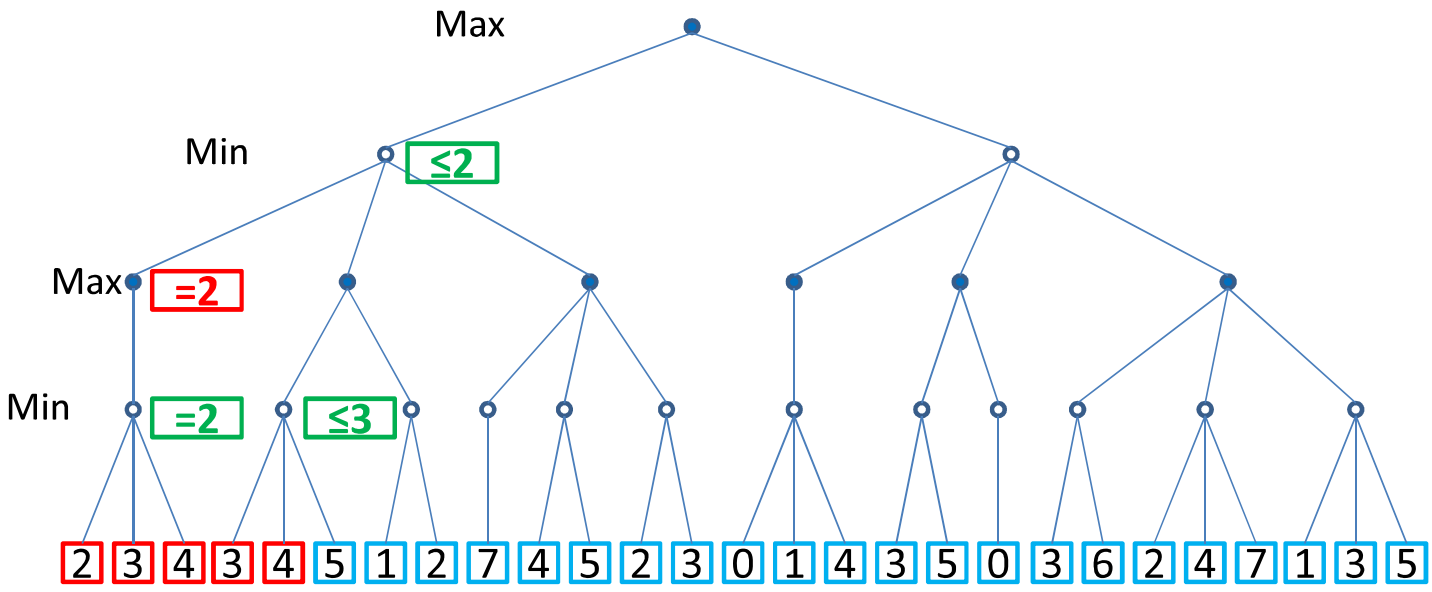

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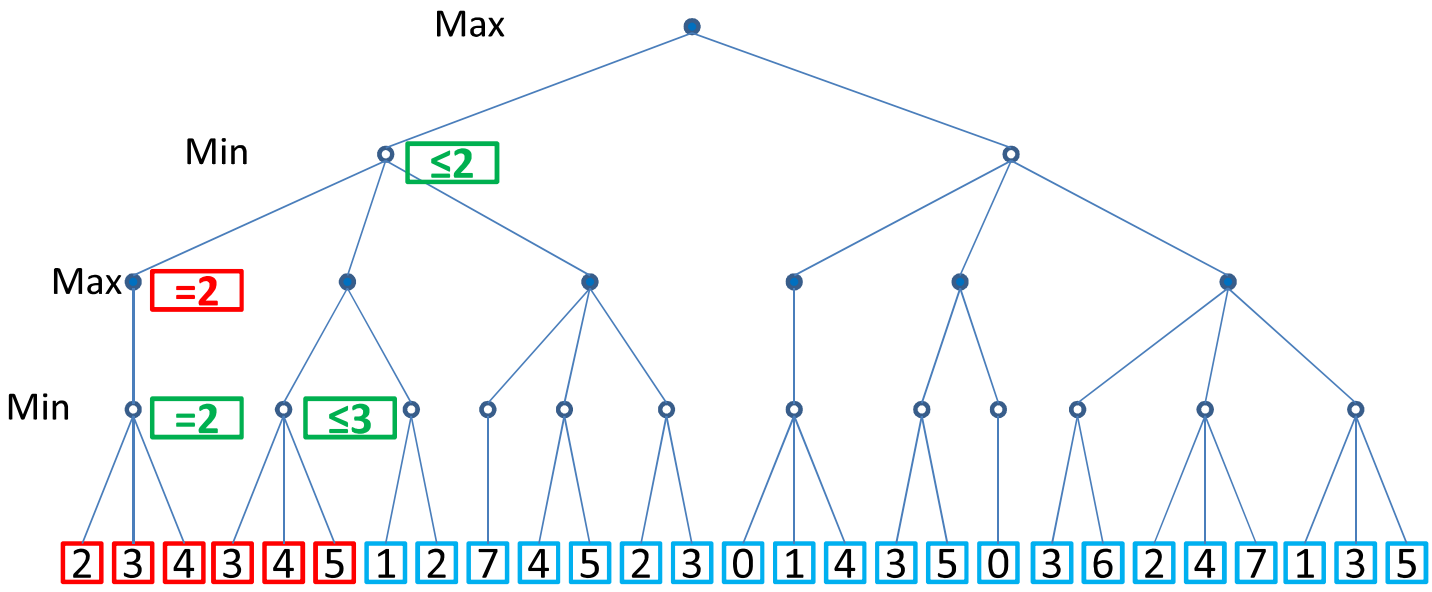

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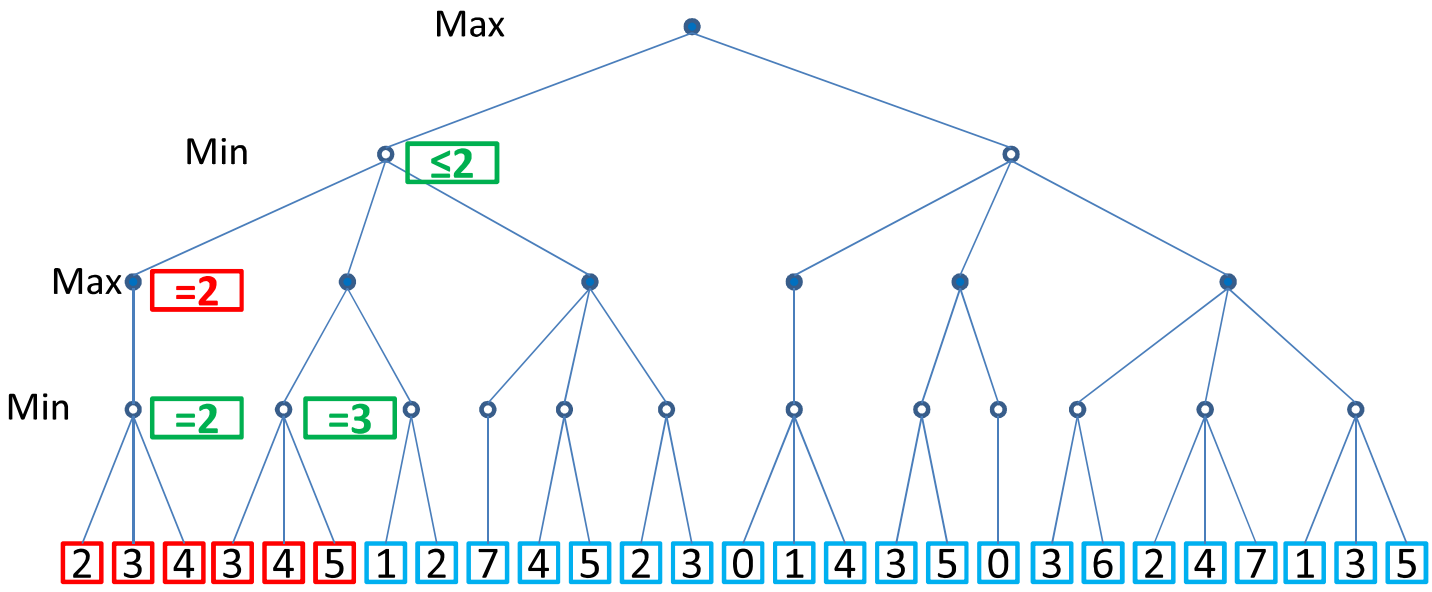

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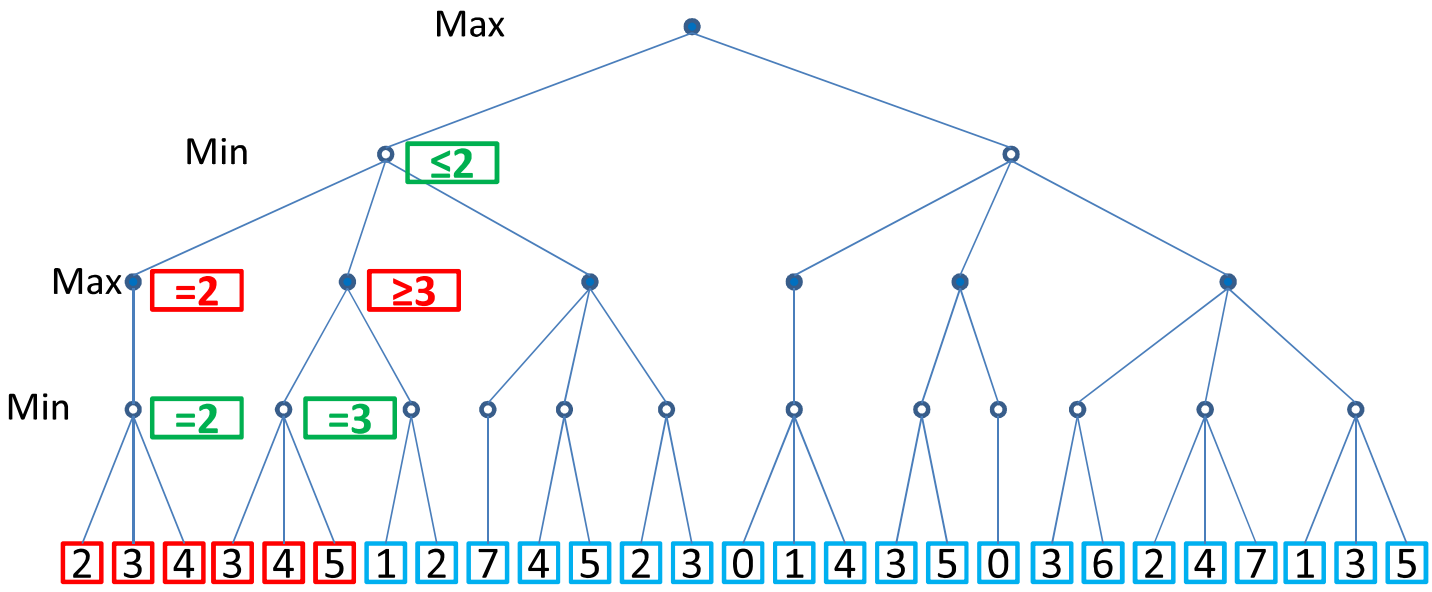

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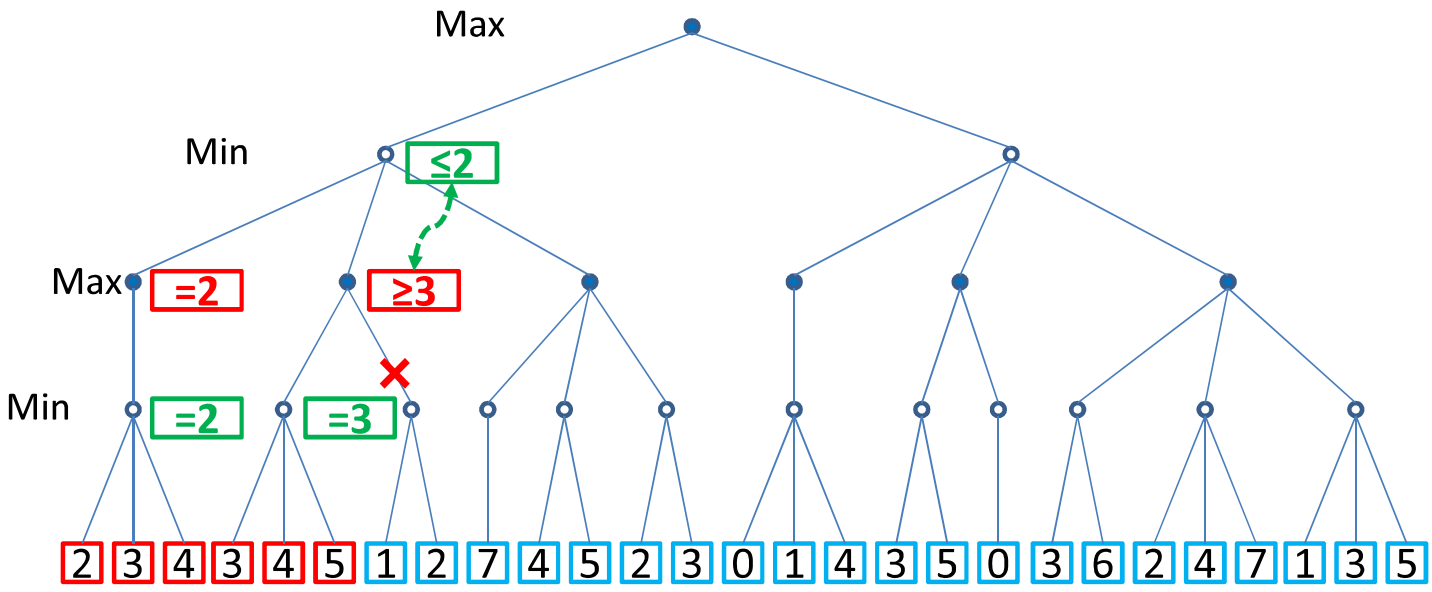

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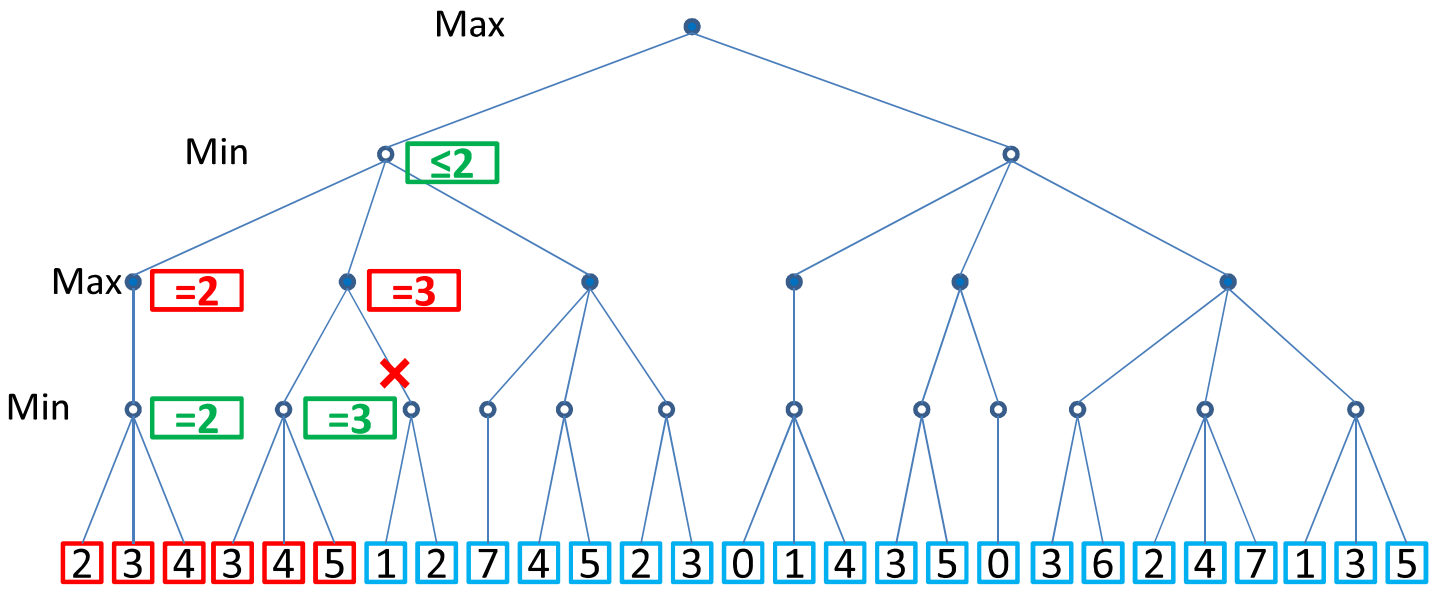

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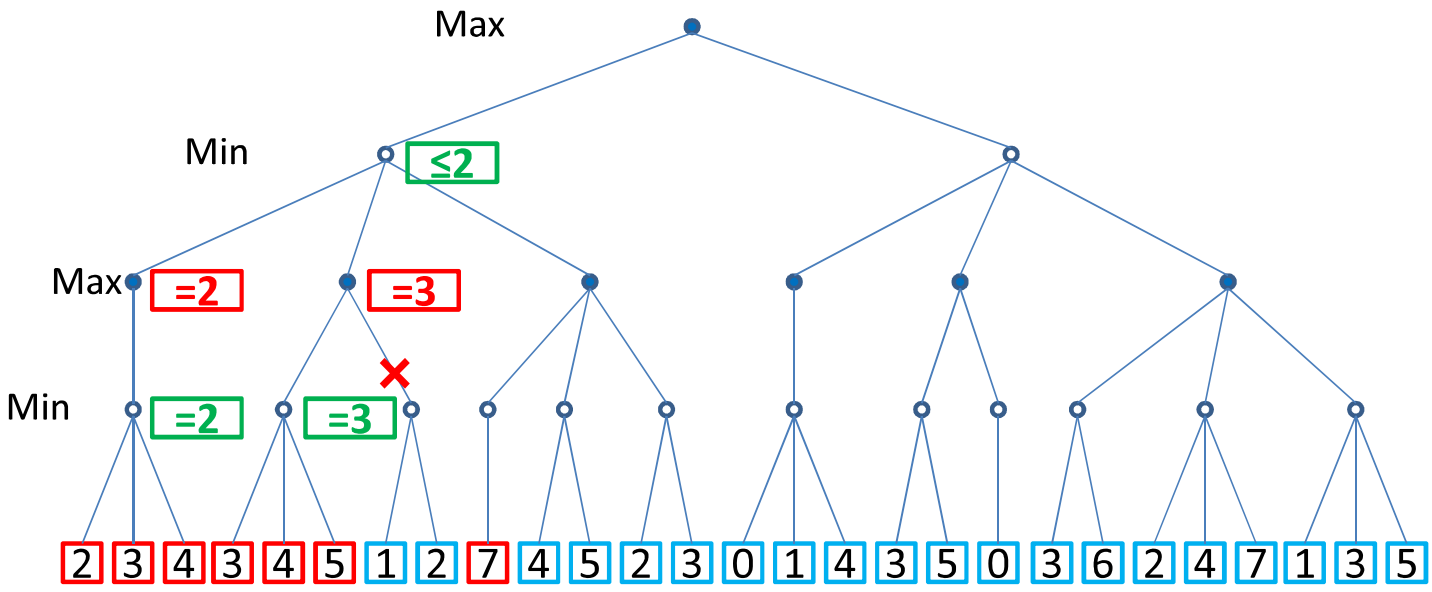

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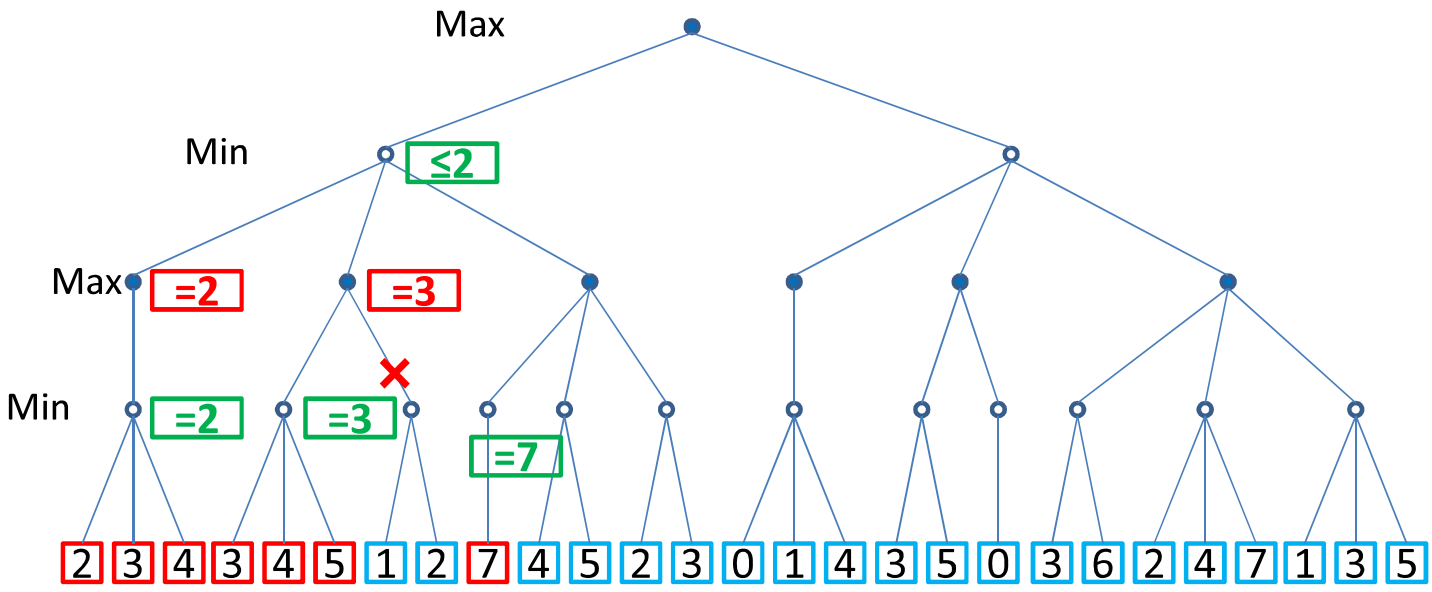

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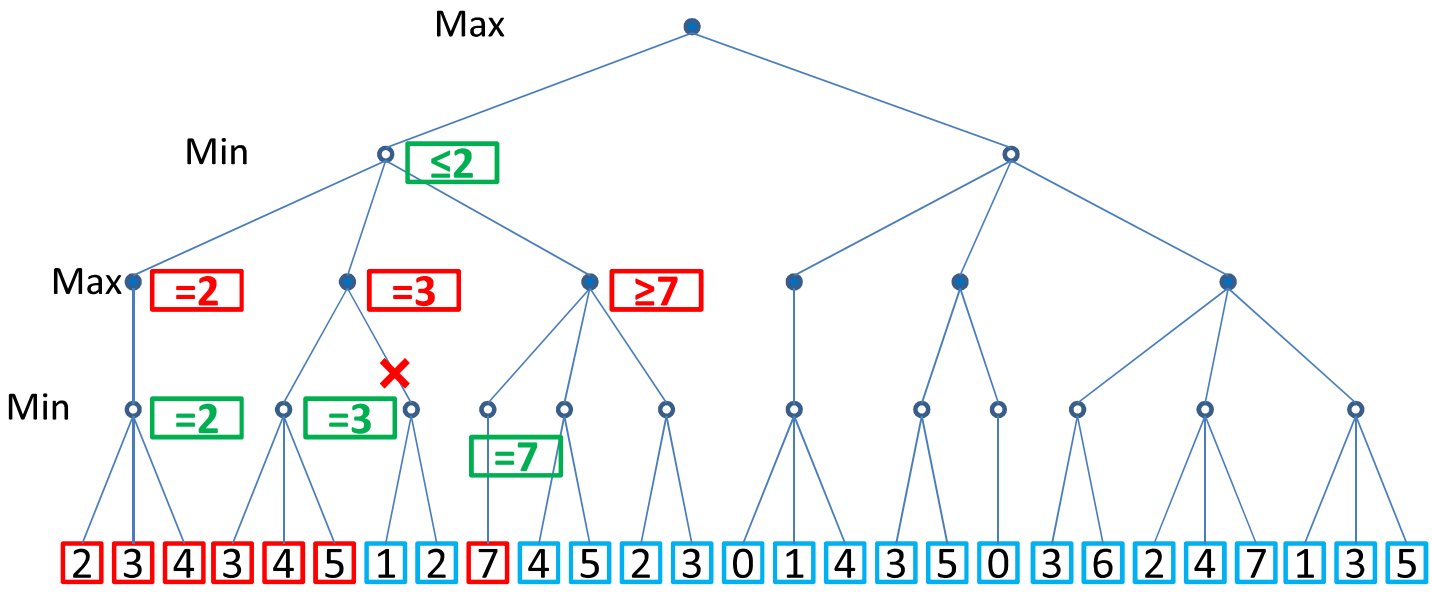

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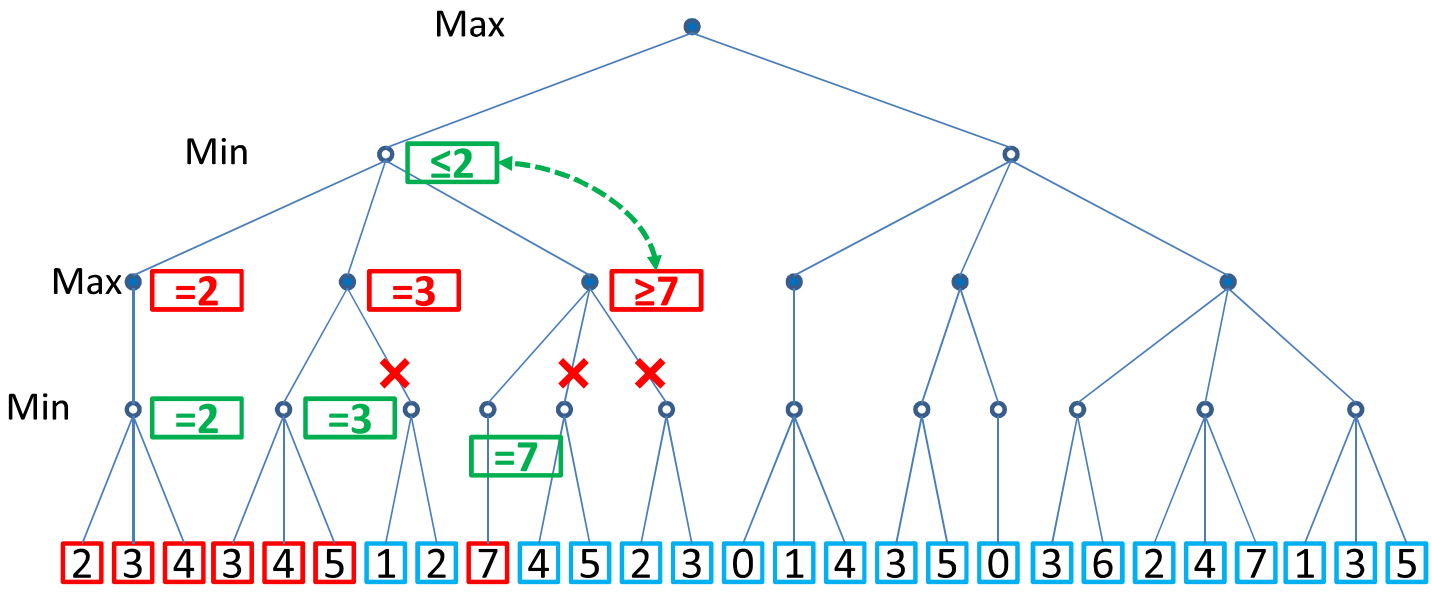

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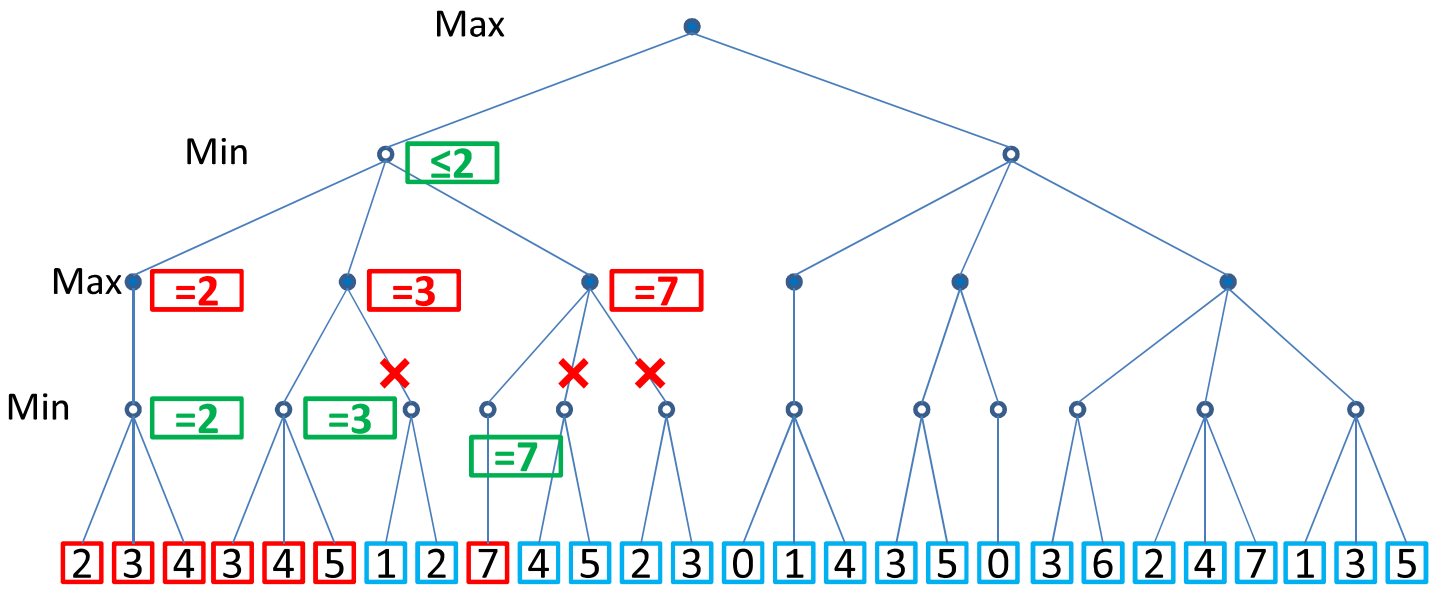

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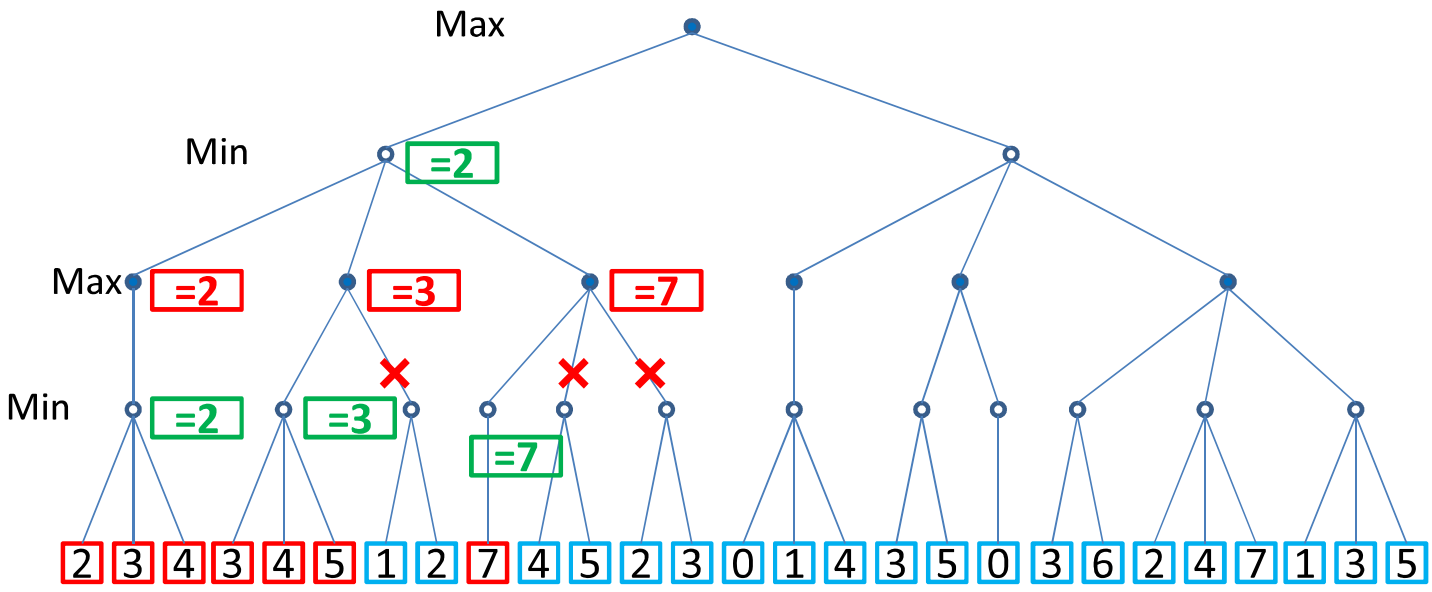

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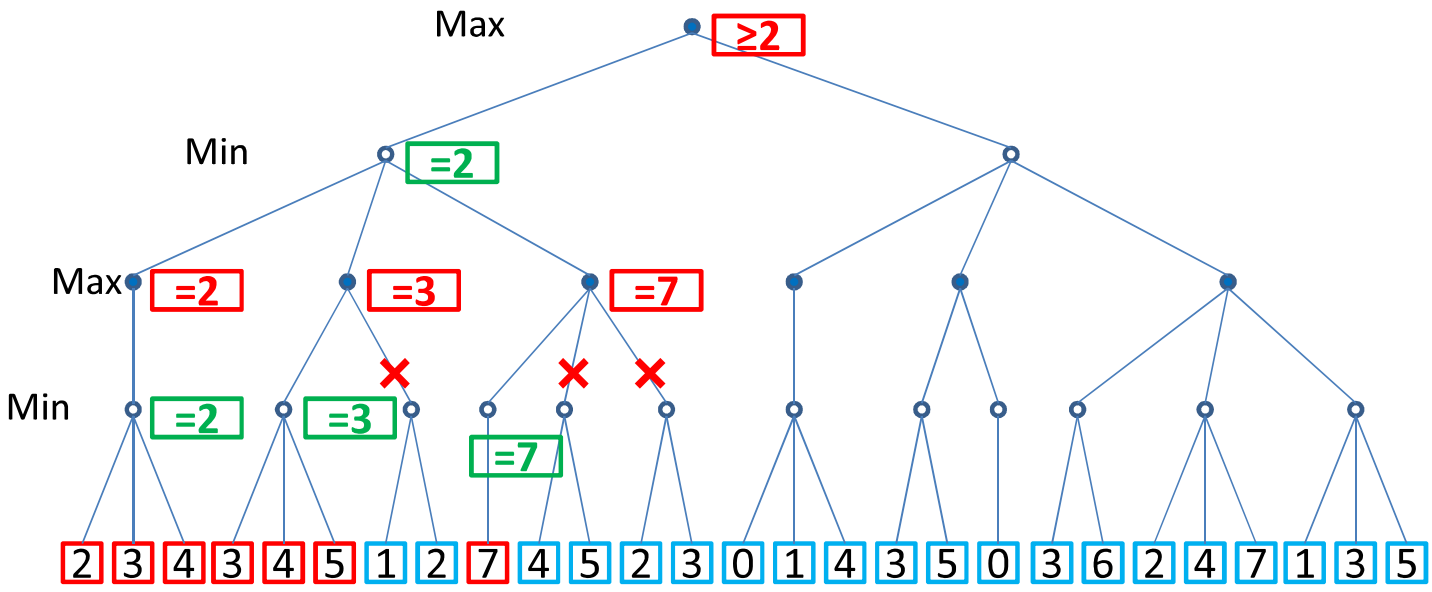

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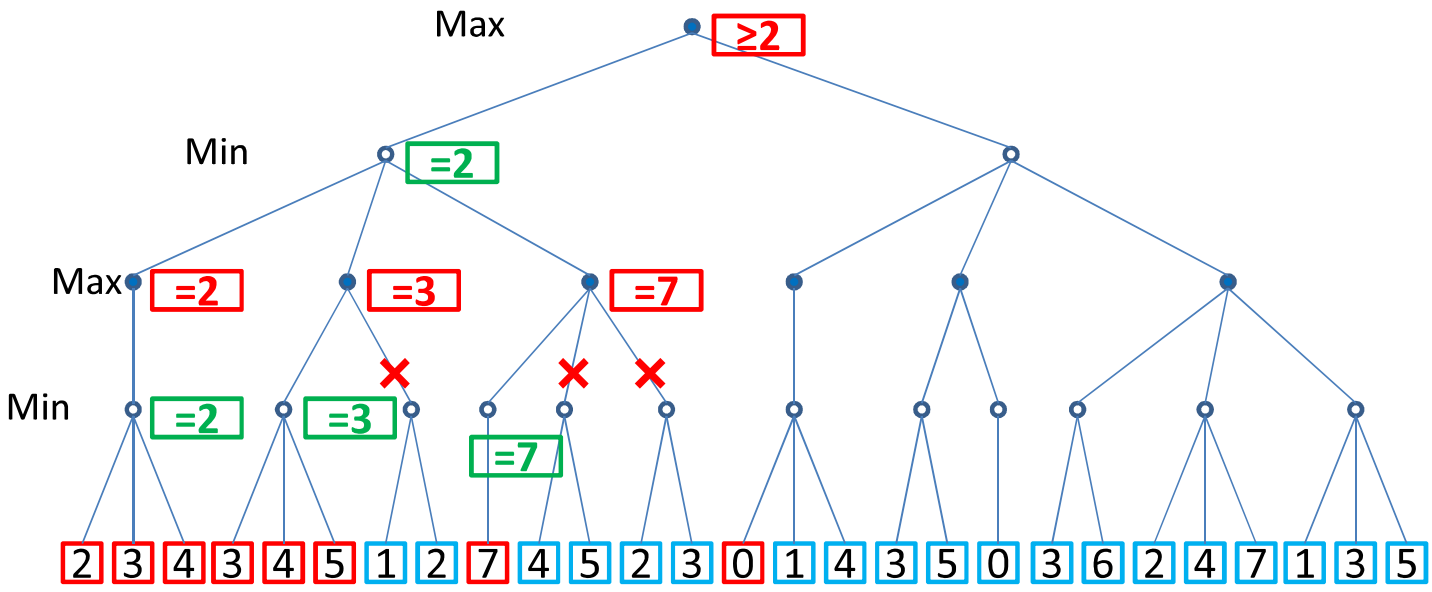

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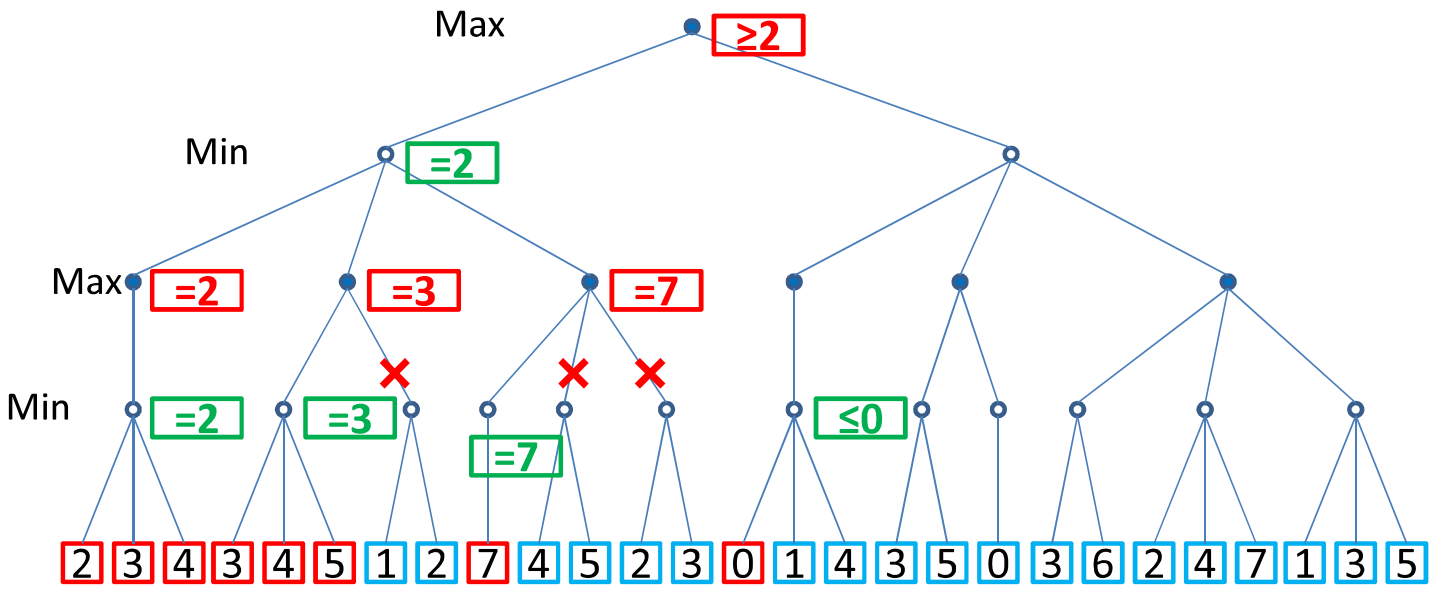

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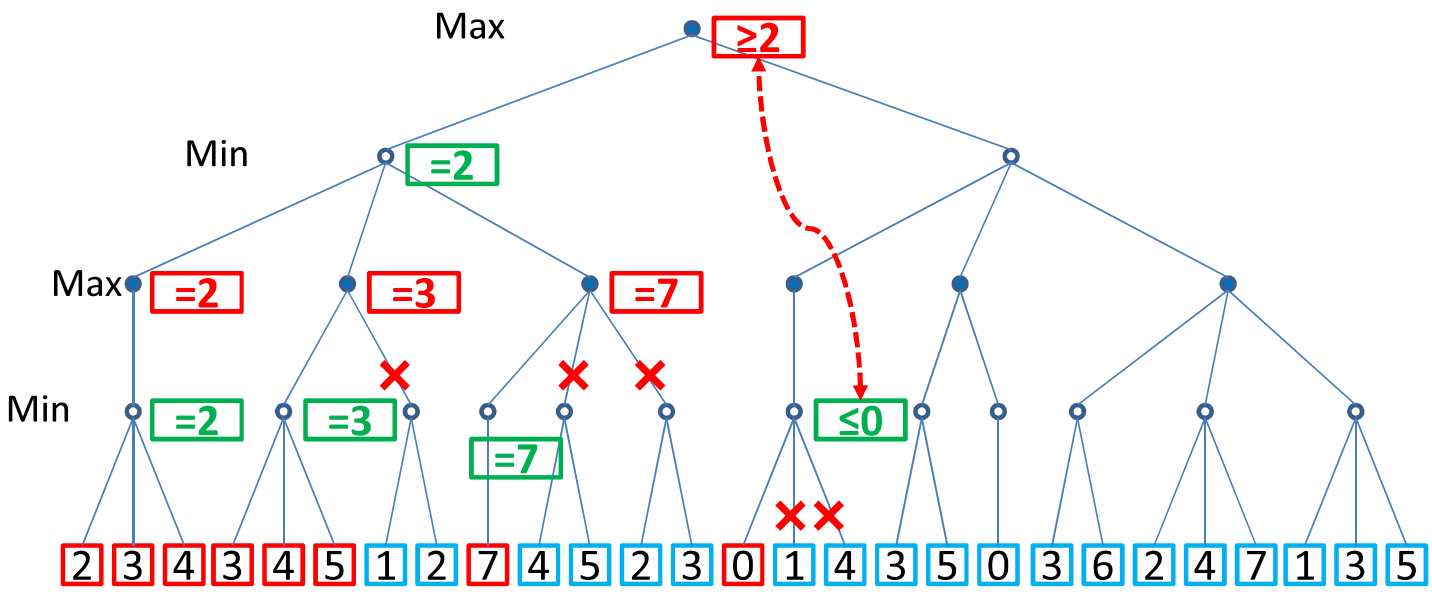

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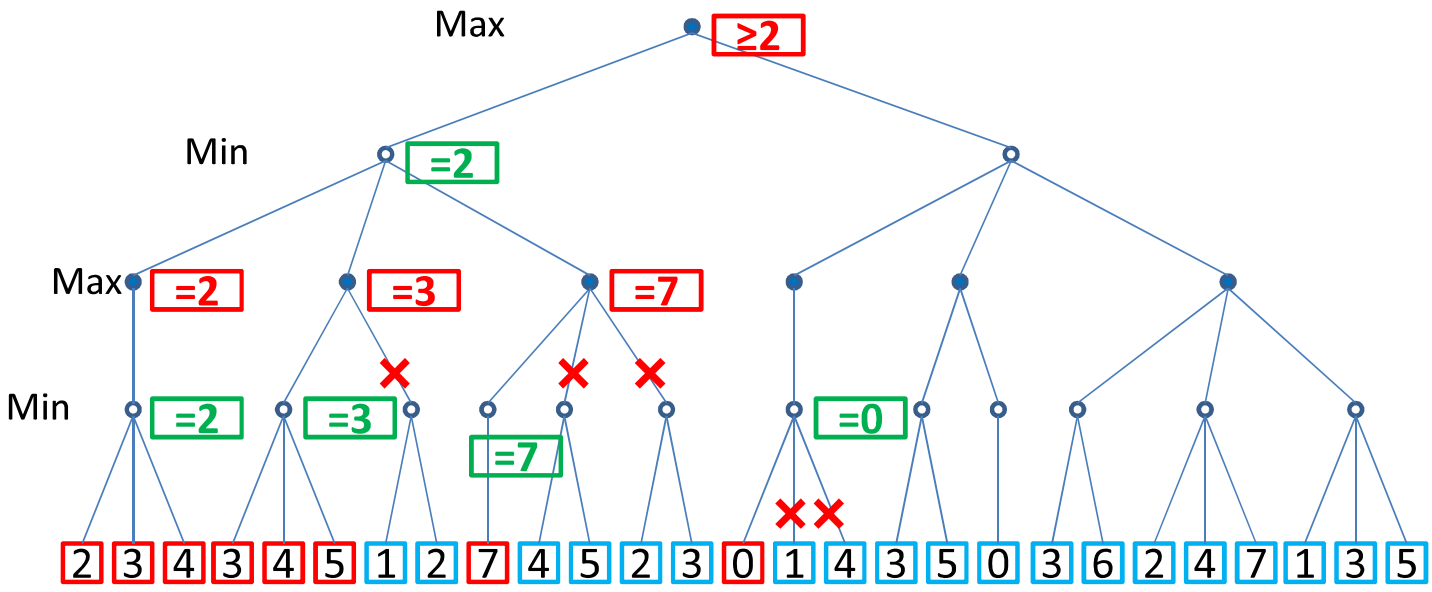

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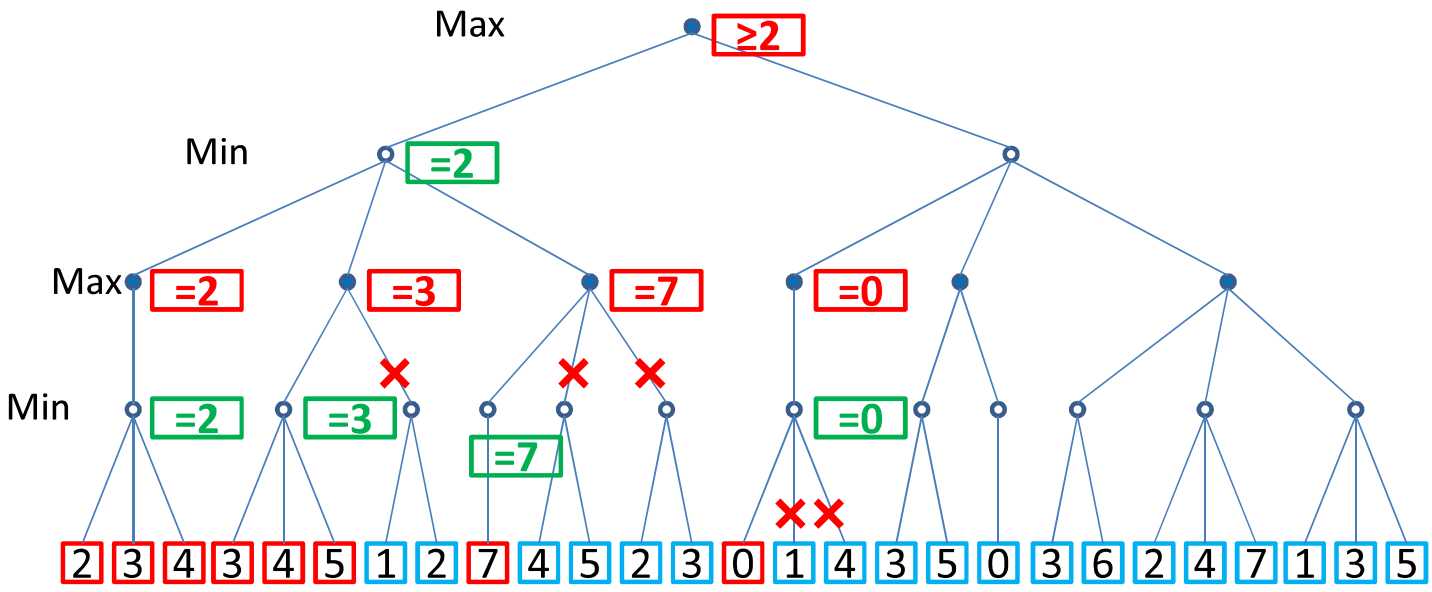

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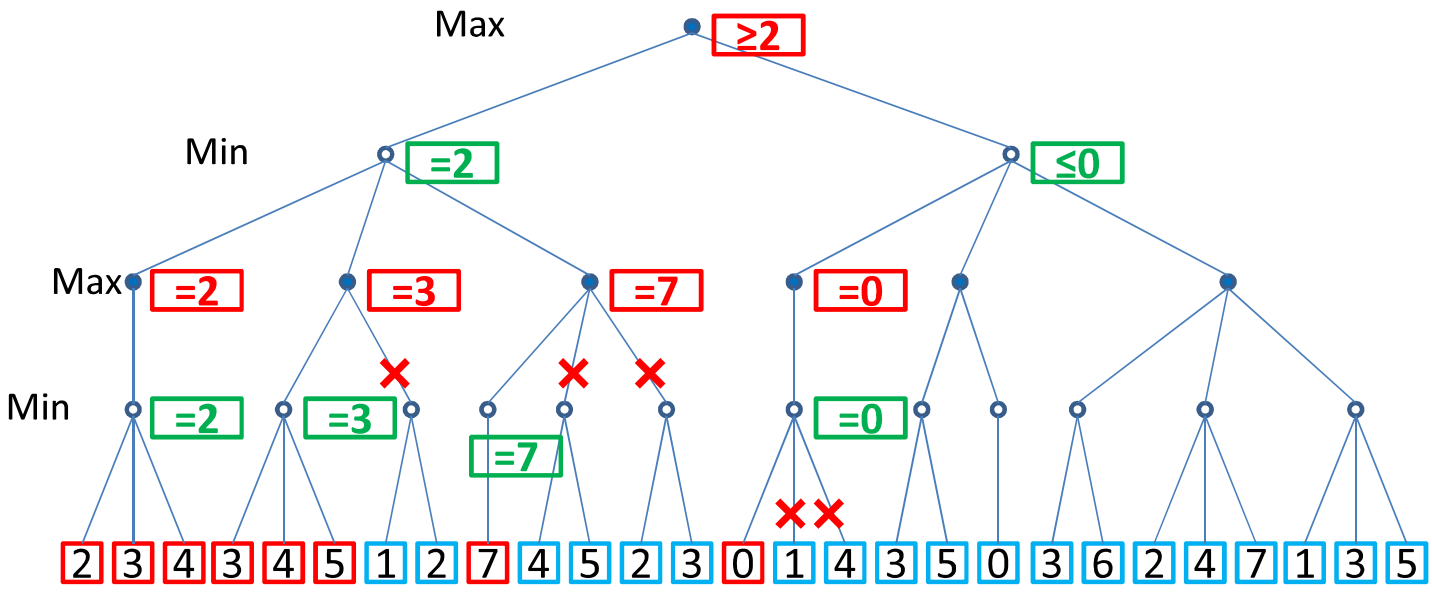

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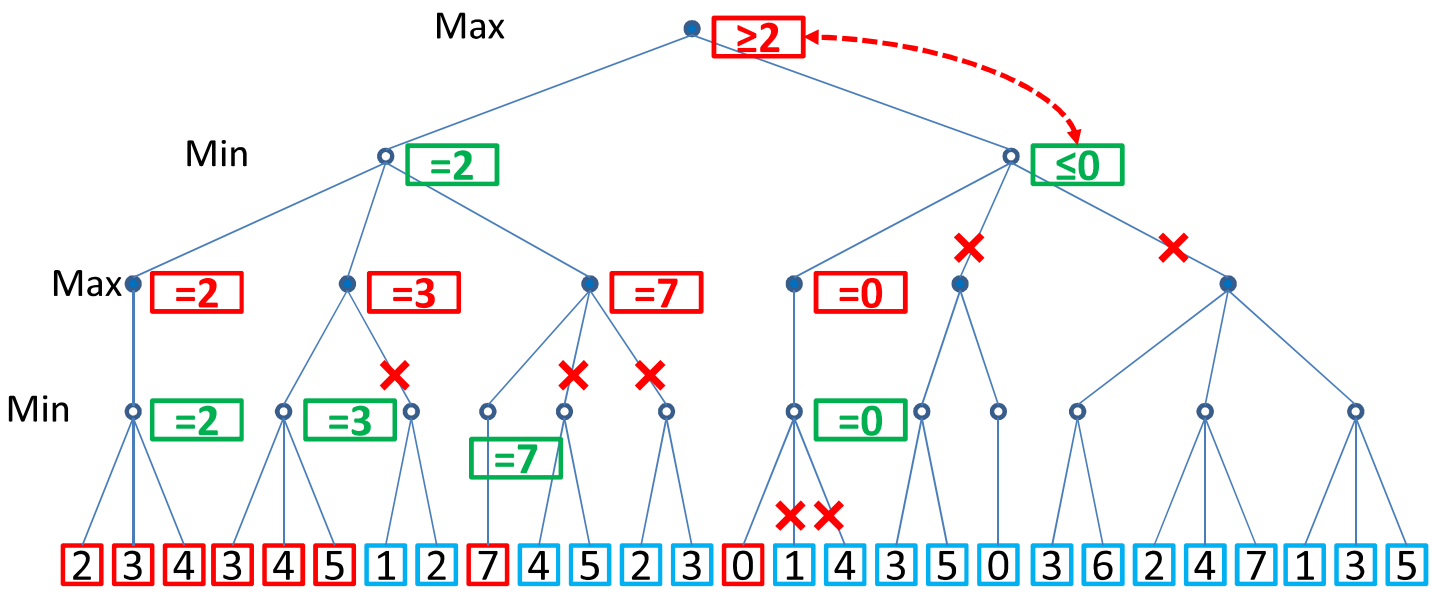

Minimax with $\alpha\beta$-Pruning

- **19 static evaluations saved**